AN INVESTIGATION INTO THE SIMULATION AND MEASUREMENT OF HIGH INTENSITY ULTRASONIC SYSTEMS

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For all those who gave everything to allow others the opportunity to follow their own choices in life.
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ABSTRACT

High Intensity ultrasound has found application in a great variety of industries and research applications, from enhanced chemical processing to industrial cleaning. The low frequencies and high power involved typically induce a phenomenon known as cavitation in the load media, upon which high pressures and temperatures can be created. The locality of these cavitation effects can be very much dependent on the internal spatial pressure distribution generated within the ultrasonic system. However, despite widespread use of such systems in industry for many years, their design is severely constrained through lack of accurate pressure measurement and subsequent simulation data. Several factors contribute to this problem: lack of hydrophone manoeuvrability in the vessel, the disruptive influence of hydrophone on the pressure profile and potential damage to the probe itself. Hence, any measurement solution that can negate these issues and lead to a more reliable simulation protocol is an attractive proposition.

This Thesis solves these problems through the implementation of a non-invasive pressure measurement system for cylindrical reactor vessels. The technique is a progression of laser interferometry coupled with Tomographic reconstruction routines. Extensive Finite Element Analysis (FEA) is conducted in order to provide a stable simulation platform for experimental validation and future virtual prototype creation. Furthermore, the possible effects of non-linear propagation on field profiles within such low frequency, high-power systems are investigated where harmonic amplitude is a prominent metric. Finally, a practical system is experimentally measured with good agreement achieved with FEA simulations. This provides a
foundation from which more exotic performance enhancing vessel designs can be launched without recourse to conventional resource intensive manufacturing processes.
CHAPTER 1

1. INTRODUCTION
1.1 Background

Low frequency high-power ultrasound has found application in a wide variety of industries ranging from food processing to pharmaceutical production. Through insonifying a target load, most commonly within a reverberant environment, it is possible to initiate the phenomenon of acoustic cavitation if conditions are suitable and pressure reaches sufficient levels. This can often facilitate the formation and collapse of microbubbles in the liquid that can subsequently produce exceptionally high localised pressures and temperatures at the bubble. Therefore, as regions of high pressure are viewed as key to the mechanisms that these systems are based on, a crucial factor in the design of these systems is accurate spatial knowledge of the internal pressure fields. However, obtaining reliable information of the pressure field in such systems is problematic at present. Conventional measurement methods are often inadequate, mainly due to the physical nature of the hydrophone device causing disruption to the spatial pressure profile. Manipulation of probe position within sealed vessels also presents challenges that can severely hinder accurate measurement. Moreover, depending on the method chosen, the measurement may also be subject to bandwidth and sensitivity issues at relatively low frequencies. The measurement device may also be irreparably damaged due to the violent forces produced during bubble collapse.

A direct consequence of poor field characterisation methods is that system designers are unable to corroborate simulation data with a reasonable degree of accuracy. This leads to an inefficient design process being adopted where many physical prototypes are constructed in order arrive at a solution that achieves the desired efficiency. Often, the solution to inefficient systems is to simply increase the input levels although,
counter-intuitively, this can actually impede the transfer of acoustic energy into load, hence reducing process output. Furthermore, many simulation techniques are either analytical in nature and do not fully take into account the extremely reverberant environment in which these systems function, or they simply cannot truly represent the acoustic conditions present within. The latter scenario refers to primarily the effects of cavitating fields and the impact non-linear propagation has on the field profile.

Ideally, the creation of a non-invasive pressure measurement system with wide bandwidth potential, high sensitivity and fine scale resolution would solve many of the challenges identified in the formation of a high-power ultrasonic system for processing applications. This measurement technique, coupled with an accurate simulation tool that is malleable to wide ranging design parameter change, offers the opportunity to design such high power systems with an improved degree of reliability and accuracy. The improved efficiency that will result will present both operational savings in terms of product output and energy input, but will also represent a significantly reduced time to deliver a system fit-for-purpose.

This Thesis addresses both the measurement issues associated with such systems and the subsequent development of reliable pressure field simulation methods. These tools are then used as a platform from which several potential system optimisation designs are presented as examples to the efficacy of this improved Virtual Prototyping approach.
1.2 Aims and contributions of the Thesis

1.2.1 Aims of Thesis

- To develop a non-invasive measurement procedure for the characterisation of high power, low frequency ultrasonic fields within sealed vessels. The technique will remove the need for probe insertion while providing accurate spatial pressure mapping with a large measurement bandwidth and sensitivity.

- Review a wide selection of high power applications featuring low frequency ultrasound, and provide an informed critique on contemporary measurement methods that may be utilised for accurate field characterisation.

- Extensively characterise the behaviour and pressure emission traits of low frequency Tonpilz transducers for employment in high power ultrasonic applications.

- Provide accurate Finite Element simulations for rapid virtual prototyping design methods for the optimisation of high power systems. These models will provide a reliable basis for a selection of design variables to be evaluated.
- Evaluate the influence of non-linear propagation on the field profiles generated in such applications and provide a means of simulating such behaviour if necessary.

### 1.2.2 Contributions to the Field of Ultrasonics

A comprehensive review of conventional field measurement techniques and their relative merits in characterising high power fields is presented. This review serves to highlight some of the aspects of high power ultrasonics that are not commonly found in the literature. Furthermore, it acts as a critique of measurement techniques in their efficacy in quantifying high power fields. An introduction to cavitation and the factors influencing it is also described.

An experimental procedure for the non-invasive pressure measurement of ultrasonic fields within sealed vessels is described and implemented. The technique features modified tomographic routines that utilise laser interferometry to map 3D pressure fields. The technique is also applicable to all non-cavitating high power fields and has potential for accurate HIFU field measurement.

An extensive transducer characterisation programme for the behavioural evaluation of Tonpilz transducers is presented. These transducers demonstrate pseudo-piston behaviour at their resonance frequency but are also subject to severe bending modes at particular frequencies. Interestingly, they also demonstrate 2\textsuperscript{nd} harmonic behaviour at nominal operational voltage levels.
An accurate Finite Element modelling methodology for simulating cell internal pressure fields has been created. The model features accurate representations of a Tonpilz transducer operating in a cylindrical test cell. Importantly, these models can be used as a rapid virtual prototyping tool in conjunction with reliable knowledge of the transmission devices employed.

An intuitive analysis is presented of the non-linear effects generated in a typical low frequency, high power system and the influence they may have on field dynamics. The merits of popular analytical models are compared with the possibility of using a numerical FE approach. The transient FE code demonstrates the potential for modelling non-linearities in heterogeneous and complicated fields.

The measurement technique and finite element methods developed in this Thesis have been used as a solid basis for further investigation into utilising alternative methods of optimising the performance of high power systems through targeted fields and phased arrays. This has lead to the success of a recent EPSRC grant application for Strathclyde to continue much of the preliminary work described in this Thesis.
1.2.3 Publications to Date Arising as a Result of this Thesis

Journal Publications


Conference Publications


1.3 Overview of Thesis

Chapter 2: High power ultrasound: Applications and Measurement

This Chapter presents a review of the common applications of high power ultrasound and of the measurement techniques available for characterising the fields involved. From this review, the role of cavitation often emerges as the main mechanism for the efficacy of each application. Furthermore, a critique of the measurement techniques assesses their suitability to characterise high power fields. Consequently, it is concluded that conventional probes will both cause a disturbance to the field while incurring irreparable damage if in a cavitating field. Therefore, an optical technique is presented as a suitable candidate for measurement of said fields.

Chapter 3: Development of a Non-Invasive Field Measurement Technique

This Chapter describes a theoretical and experimental investigation into the use of laser interferometry and computer tomography for the characterisation of pressure fields. Firstly, the application of the acousto-optic effect combined with laser interferometry for pressure quantification is described. Next, a synchronous scanning algorithm for implementing simple computer tomography is developed and verified on air coupled ultrasonic fields. Finally, the novel modification of this scanning procedure for mapping internal vessel pressures is presented.

Chapter 4: Finite Element Modelling

This Chapter presents an accurate model of cylindrical pressure vessels incorporating Tonpilz transducers. First, the behaviour of the Tonpilz transducers is thoroughly experimentally characterised and shows excellent agreement with the simulations.
Next, the modelling is extended to include the cylindrical test vessel with a water transmission load. Finally, the influence of two types of measurement probe on pressure profile is simulated in the FE environment, with disturbance of the pressure field clearly evident in both.

**Chapter 5: Non-Linearity Investigation**

This Chapter investigates the possible non-linear effects generated within the transmission loads of these high power systems. An intuitive overview of classical acoustic non-linearity in relation to wave propagation is outlined. Several popular models for non-linear propagation are described with the relative merits of each noted. Next, the application of transient FE analysis for solving complex non-linear problems, such as those involving reflective boundaries, is presented. An experimental study isolates sources of non-linearity and quantifies their influence approaching the cavitation threshold of the load. It is concluded that no significant non-linear effects are generated in such systems before the initiation of cavitation. Therefore, linear systems can be used as a representative tool for the simulation of fields in similar test cells.

**Chapter 6: Non-Invasive Measurement of Ultrasonic Fields in Sealed Vessels.**

This Chapter demonstrates the application of the non-invasive measurement technique for the accurate characterisation of pressure fields generated by 33 and 40 kHz systems. The experimental measurements show good correlation with the theoretical models with possible reasons for discrepancies proposed. Several factors affecting field profiles are investigated with experimental results provided. Furthermore, variations of high power systems are simulated in an attempt to optimise with
performance with examples presented. It can therefore be stated that the accurate mapping of the fields within sealed vessels can be achieved through a combination of the acousto-optic effect, laser interferometry and modified computer tomography routines.

Chapter 7: Conclusions and Future Work

The Thesis concludes with a review of the material contained within all previous Chapters and presents a host of practical suggestions for furthering the knowledge in this field, with the initiation of future research programmes within Strathclyde a key goal.
CHAPTER 2

2. HIGH POWER ULTRASOUND: APPLICATIONS AND MEASUREMENT.
2.1 Introduction

Since the pioneering work published by Lord Rayleigh on the theory of sound in 1896 [1], ultrasound has been utilised in a wide variety of applications. The prolificacy of ultrasound in fields such as SONAR [2], Non-Destructive Testing (NDT) [3] and biomedical imaging [4] has produced an abundance of literature since the beginning of the First World War, where its potential as a viable means to detect submerged objects became apparent. The innovative work conducted during wartime by scientists such as Langevin, continued unabated in post war years with attention turning away from large scale inspection of the oceans to small scale probing of specific regions of interest. The concept of ultrasonic metal flaw detection was first suggested by Sokolov in 1928 although the absence of suitable equipment to generate and receive short pulses until the early 1940’s resulted in very poor resolution. By 1948 researchers in the United States and Japan were independently investigating the potential of ultrasound as a medical diagnostic tool. Previously, the use of ultrasound in medicine had initially begun with rudimentary applications in therapy, e.g. tissue breakdown, rather than imaging. This destructive ability of high intensity ultrasound was observed by Langevin when he noted the death of schools of fish in the sea and pain induced in the hand when placed in a water tank insonated with high intensity ultrasound. It was around this time that development work into the use of high intensity ultrasound for industrial processes began.

High power ultrasonic fields are used in a number of diverse application areas, which are described extensively in Section 2.3. The definition of ‘high power’ or ‘high intensity’ can be thought of as ultrasonic fields that directly influence a process, i.e.
induce a permanent physical change in a target object or region through exposure to vibrational energy. Quite often, this involves the utilisation of cavitation, which will be discussed in Section 2.2, to produce some desired effect. Generally, high power applications operate using low frequencies (10 kHz to 500 kHz) and relatively low power diagnostic and inspection applications function at higher frequencies (from 2 MHz in NDT applications, to 100 MHz in some biomedical diagnostic scanners). There are, of course, exceptions to this categorisation, such as SONAR, where intense power levels are involved in conveying information, and High Intensity Focussed Ultrasound (HIFU) applications, which will be discussed in Section 2.3.2.3, where high frequency fields are used to destroy tissue.

Measurement methods for these high power fields are important for safety and process efficiency reasons yet, currently, there are very few well-documented and reliable measurement methods available. Initially, due to the harmful tissue damaging effects experienced in the early use of ultrasound in medicine, the development of standard measurement techniques required to quantify medical equipment output levels was quick and comprehensive. Hence, standard techniques have been long established for the measurement of low power ultrasound in the medical field [5, 6]. Moreover, measurement of the output levels from medical devices is made easier by the fact that many systems operate in pulsed or tone burst modes, providing the assumption that free-field conditions applies. This is generally not the case for high power systems, with the exception of lithotripsy, which normally operate in continuous wave (CW) mode and within environments that are extremely reverberant i.e. cylindrical test vessels several wavelengths in length. Given this, there is an increasing demand for knowledge of the spatial distribution of pressure fields within
these environments, for example in applications such as ultrasonic cleaning [7] regions of the field are known to clean better than others. It is therefore desirable to have a reliable way of quantifying these spatial variations of pressure without adversely disrupting the dynamics of the system itself. However, before expanding on contemporary methods it is important to understand the current body of knowledge associated with field measurement and, in particular, high power fields. This chapter will review some of the more common applications of power ultrasonics; provide an introduction to other high intensity applications, such as sonochemistry and HIFU; outline the main mechanisms at work in many high intensity ultrasonic applications and, finally, discuss the merits of current field measurement techniques.
2.2 Cavitation

Acoustic cavitation is simply acoustically induced bubble activity. The activity of the bubble itself is not the source of many of the advantageous effects (or disadvantageous effects, depending on the application) of cavitation; bubble activity can be merely the oscillation in radius size caused by an incident sound wave. It is the subsequent collapse of these bubbles into a volume considerably less than their original size that generates the dramatic effects associated with acoustic cavitation.

This section serves an introduction to some of the more common effects and parameters which influence the level of cavitation in a high power ultrasound field, in addition to discussing the two categories of cavitation and the thresholds between them. For a more extensive analysis of cavitation; the causes, effects and applications, the reader is referred to Leighton’s impressive text [8]. A useful ‘broad-stroke’ tutorial on cavitation by Apfel is also recommended [9].

2.2.1 Introduction

An ultrasonic field in a liquid may cause the expansion of microscopic bubbles present during the negative cycle of the propagating wave. If the amplitude of the acoustic waves is sufficient, these bubbles will undergo several rapid expansions before reaching a critical radius, $R_{MAX}$, at which point the bubble suffers a violent collapse [10].

$$R_{MAX} = 2.3R_O$$  \hspace{1cm} (2.1)

where: $R_O = \text{Bubble equilibrium radius}$. 
Within the region of collapse, several spectacular effects are likely to occur, including an internal bubble temperature of 3000 K and pressure shockwave emission reaching 6 GPa [11]. Bubble motion of this nature is called transient cavitation or, contemporarily, inertial cavitation and the entire cycle can occur to the seed bubble several times before it fragments. When the amplitude of the acoustic wave is below the threshold to induce inertial cavitation, the alternative motion of the bubble is referred to as stable or non-inertial cavitation. This variety is non-destructive and long-lived in comparison. Despite exhibiting none of the impressive effects of inertial cavitation, non-inertial cavitation can provide valuable information on the properties of the liquid in which it exists, in addition to revealing some of the subtleties associated with bubble motion if suitably studied. Figure 2.1 shows the possible outcomes for a bubble excited by an ultrasonic field.

2.2.2 Inertial Cavitation Threshold

The likelihood of inertial cavitation occurring in a high power ultrasonic system is an important assessment criteria when evaluating the system effectiveness. Some excellent literature exists analysing this problem, with attempts to define clear boundaries between inertial and non-inertial cavitation (not trivial due to the scope of parameters involved) culminating in a series of prediction charts in Apfel’s publication [10]. As mentioned, due to the dubiety that exists between inertial and non-inertial cavitation these graphs are meant to taken semi-quantitatively. Nonetheless, they serve as an excellent guide for predicting the type of cavitation that can be expected if the operational parameters of the system are known. The following sub-sections serves merely as an intuitive guide on how some of the common
parameters involved in typical ultrasonic systems, and used in Apfel’s prediction charts, can influence the level and type of cavitation generated.

2.2.2.1 Frequency

One of the most versatile parameters at one’s disposal when attempting to manipulate acoustic cavitation is the frequency of the incident sound wave. Frequency is crucial to the expansion process of a bubble and has a profound effect on the pressure threshold for both the onset of inertial and non-inertial cavitation. Taking one cycle of an acoustic wave where the pressure amplitude is negative for one half wavelength hence, the pressure of the load medium decreases and the forces in and around the bubble are no longer in equilibrium. This results in the bubble expanding for the

![Diagram](image_url)

Figure 2.1. A typical seed bubble being excited by an acoustic wave. The bottom path demonstrates bubble behaviour when acoustic amplitude is relatively low: non-inertial cavitation. The top path when acoustic amplitude is great enough to cause the bubble to expand past $R_{\text{MAX}}$: inertial cavitation.
duration of the negative cycle, with the rate of expansion dependent on the acoustic pressure: greater pressure amplitude; more rapid expansion and vice versa. If the frequency of the acoustic signal is increased the period of exposure in which the bubble is subject to negative pressures decreases. Therefore, there is a subsequent reduction in $R_{\text{MAX}}$ and the acoustic amplitude must be increased to induce cavitation, assuming this is the objective. Given this, it is easy to understand why many power ultrasonic systems operate at relatively lower frequencies (less than 100 kHz), and why the majority of medical diagnostic systems pose relatively little danger in terms of tissue damage induced from inertial cavitation.

### 2.2.2.2 Acoustic pressure

The acoustic pressure amplitude of the sound field at the bubble is another critical threshold parameter. It is known that increased acoustic pressure can enhance the expansion rate of a bubble. However, the crucial factor in this instance is not the input amplitude to the transducer, but more the local acoustic pressure in the region of the bubble. While it is relatively easy to monitor the vibrational amplitude of the active face, obtaining the pressure distribution at the bubble is not so simple. It is therefore important that accurate spatial knowledge of the pressure field exists for a system designed to elicit cavitation, as this highlights the regions of the load in which inertial cavitation is more likely to occur. As an addendum, it is clear that accurate field measurement techniques are therefore imperative to the design of such systems.

### 2.2.2.3 Surface Tension

The accepted description of bubble behaviour due solely to the influence of surface tension was formulated by Blake in 1949 [12] and is known as the ‘Blake threshold’.
It defines the threshold pressure required for expansion relating to overcoming surface tension only.

Given a bubble at rest in a quasi-static medium, the pressure within the bubble, $P_i$, is greater than the pressure in the liquid, $P_l$, immediately outside as a result of surface tensions forces. The force that balances this excess internal pressure and keeps the bubble intact is the surface tension. Figure 2.2 illustrates the forces acting on a stable bubble, where surface tension, $P\sigma$, acts radially inward and internal pressure, $P_i$, acts radially outward. Therefore, if it were possible to split the bubble exactly in half, the excess internal pressure would force the two halves apart.

Using this description of pressure forces, if the pressure in the liquid outside the bubble is increased this would aid surface tension and force the bubble to contract. Conversely, if $P_l$ is decreased, but remains positive, the bubble will expand until a new equilibrium is reached. The bubble will remain stable throughout. However, if the liquid pressure is decreased further and becomes negative it will begin to counteract the surface tension. Assuming the bubble has a sufficient initial radius, the pressure balance cannot be maintained and the bubble grows explosively i.e. becomes unstable (inertial cavitation). As it is impossible to maintain negative pressures for more than a few microseconds, the bubble will subsequently collapse. As a result, Blake’s threshold pressure does not describe explosive growth, i.e. dynamic frequency dependent conditions, but it is valid for predicting the onset of explosive growth in small bubbles, where surface tension forces dominate.
It should be noted that for explosive growth to begin these bubbles must be greater than some critical size, since as \( R_O \to 0 \) the surface tension pressure increases without limit, \( P\sigma \to \infty \). Hence, the minimum pressure in the liquid required for growth is a function of the bubble radius, \( R_O \), and is therefore [8]

\[
P_L = P_V - P_B = P_V - \frac{4\sigma}{3} \sqrt{\frac{2\sigma}{P_o + \frac{2\sigma}{R_O} - P_V}}
\]

where: 
\( P_B \) = Blake Threshold Pressure. 
\( P_V \) = Vapour pressure within bubble. 
\( \sigma \) = Surface Tension (force per unit length acting perpendicular to one side of a straight line in a liquid surface). 
\( P_O \) = Ambient pressure.

For conditions where surface tension dominates all others, Equation 2.2 can be reduced to [8]

\[
P_B \approx P_o + 0.77 \frac{\sigma}{R_O}
\]
2.2.2.4 Fluid Viscosity

One other factor that has an influence of the cavitation threshold of a system, albeit of lesser significance than the characteristics of the acoustic signal or surface tension, is the viscosity of the fluid. Fluid viscosity alters the balance of forces depicted in Figure 2.2, in that it slows the expansion time of the bubble, and hence, reduces $R_{MAX}$. This results in higher cavitation threshold pressure amplitudes for fluids of higher viscosity. However, for low frequencies and high driving pressures this delay in expansion time is minimal compared to the overall negative portion of the acoustic cycle. Therefore, viscosity only becomes a major factor when the frequency approaches 1 MHz or a highly viscous fluid is used as the load medium e.g. light machine oil [10].

![Figure 2.2. Forces acting on a single bubble under stable conditions i.e. no acoustic wave present.](image)
2.2.2.5 Bubble Size

Of all the considerations involved in whether a bubble cavitates and under which regime it does so in, the initial radius of the bubble is subject to the least control but, at times, can prove to be the most significant variable. If the equilibrium radius is too small, then surface tension forces prevent the initial sudden growth and inertial cavitation does not occur. If it is too large, then it is likely the bubble will grow but not at a fast enough rate to concentrate the energy upon collapse to generate suitably impressive effects. Therefore, there exists a critical range in which the bubble must lie if it is to nucleate inertial cavitation in a given sound field: the lower the frequency and higher acoustic pressure, the wider the range. This range, or threshold, for inertial cavitation is plotted as a convenient graph for several frequencies and is based on the calculations established in [13,14]. This represents a more comprehensive set of prediction charts when used in conjunction with those in [10].

2.2.3 Minimising Cavitation

Often, inertial cavitation is an unfortunate consequence of the operating conditions that an ultrasonic system must adhere to in some applications, e.g. requirement of a particular frequency in medical diagnostics. When this is the case, steps can be taken to limit the possibility of cavitation through manipulating the same parameters, in addition to other methods, that are often used to optimise cavitation. From the factors mentioned already in this Section, increasing the acoustic frequency and shortening the excitation time reduces the window for expansion, while decreasing the acoustic amplitude lessens the expansion rate of the bubbles. Degassing the load medium before exposure to ultrasound is also a convenient method of drastically cutting down
the number of bubbles that have the potential to nucleate inertial cavitation. Moreover, by filtering the liquid any impurities (particles) that may provide a nucleation site or crevasse for trapped air can be removed. A more esoteric solution to the problem is to place the transmission fluid under compression therefore creating, what is in effect, a positive pressure offset. This subsequently negates the negative cycle of the acoustic wave in terms of bubble expansion potential, preventing the bubbles from reaching the minimum radius required to cavitate. Finally, it may be possible to limit cavitation by keeping the temperature in the transmission medium as relatively low as possible [15, 30].
2.3 High Power Applications

There are effectively three strands of ultrasonics research; power ultrasound, sonochemistry and diagnostic applications. This work is concerned only with the first two, which are readily interchangeable due to the reliance on predominately low frequencies and cavitation in the vast majority of applications. Indeed, these two disciplines will be the focus for this Section. Despite being beyond the scope of this work, low power, high frequency ultrasound for inspection and process monitoring, particularly in recent years, has an important role to play in many industrial applications in conjunction with power ultrasonics.

2.3.1 Sonochemistry

Sonochemistry is a burgeoning discipline within the ultrasonics community, although it would be inaccurate to describe it as a product of ultrasound research alone as its roots are more firmly planted in the sciences of chemistry and metallurgy. Indeed, the effects of power ultrasound in cleaning baths were the first exposure that scientists had to the possible benefits of acoustic energy in chemistry, when the influence on glass submersibles within these baths was noted. Given this, the term sonochemistry is often used to describe the effect of ultrasonic sounds waves on chemical reactions, which encompasses the majority of processing applications involving power ultrasound. Polymer chemistry and synthesis were initially the prime focus of power ultrasound techniques in chemistry [16]. However, more recent uses for ultrasound in chemistry have been found that are distinct from synthesis and polymer chemistry, such as; material science (new catalytic materials, improved extraction of metals), biotechnology (modification of enzyme and cell activities, used in the food industry –
section 2.3.1.1) and environmental protection (both biological and chemical, e.g. water and sewage treatment – section 2.3.1.2).

It has been established that power ultrasound derives many of its benefits via cavitation bubbles, with the production and intensity of the cavitational effects decreasing as frequency increases (Section 2.2). As the mechanical and chemical effects of cavitation are considered important for the success of sonochemistry, it is advantageous to maximise these effects by limiting the frequency range to below 50 kHz. Despite the use of higher frequencies (approximately 1 MHz) in many aspects of sonochemistry, for example oxidation processes in food processing [17], this Thesis will limit the discussion to techniques involving high power, low frequency ultrasound within sonochemistry. In particular it will focus on the areas of food technology and water treatment. For an extensive review of all aspects of sonochemistry, the reader is directed to [28, 12] and to [18, 19, 20] for its future potential as a viable technology on an industrial scale.

2.3.1.1 Food technology

There has been increasing interest in the use of ultrasound in the food industry for many years, with applications including: particle size control, process tomography, determination of material properties, monitoring of shelf life and preservation enhancement becoming common. However, many of these applications do not rely on power ultrasound as the main antagonist and are considered to be more in the diagnostic spectrum of ultrasonic applications. Nonetheless, high power ultrasound is fast becoming a useful tool when attempting to favourably alter the characteristics of a variety of foods in a ‘clean’ manner as they undergo processing. Increased demand
from consumers for methods of food processing that have a reduced impact on nutritional content, has stimulated the use of ultrasound coupled with standard sterilisation and pasteurisation methods, for microbe inactivation. Power ultrasound in conjunction with thermal and chemical techniques has been shown to reduce the numbers of many bacteria such as Salmonella and E. coli [21]. Other beneficial uses of power ultrasound in the food industry include; sterilisation, extraction of tea solids from leaves, tenderising of meat products, assisted crystallisation (freezing), degassing through numerous bubble collapses induced by cavitation. An outstanding treatment of all aspects of ultrasound in food processing is provided in [22].

2.3.1.2 Water & Sewage Treatment

Perhaps one of the most beneficial applications of ultrasonic processing for society is the potential for its use in the water and sewage treatment. The destruction or transformation of organic pollutants and the removal of biological contaminants are the fundamental objectives of investigations involving ultrasound. Until recently, it was thought power ultrasound would be too expensive as a viable technology to use for water treatment on an industrial scale. This was based on the direct scale up of power consumption in small-scale laboratory experiments. However, recent research has suggested that the decontamination of water through ultrasonic techniques in conjunction with other treatments may be feasible when applied to flowing systems [23].

Regarding water treatment, two examples of removing biological contamination from the water have been implemented on a large scale basis; inactivation of plankton clogging filters in water distribution systems; and the destruction of algal blooms
The former demonstrated satisfactory results in plankton inactivation using economic power levels and a flow through system, while the latter demonstrated that ultrasonic treatment offers the potential to not only kill the micro-organism but also severely restrict its reproductive ability. In sewage sludge treatment, ultrasound is often applied as pre-treatment to enhance the time-consuming and inefficient conventional processes, without the requirement for relatively large amounts of power to be transmitted [25, 26]. The benefits of ultrasonic pre-treatment with application to contaminant removal has also been considered for other areas, such as distillery wastewater [27], although a conventional ultrasonic bath was used in the experimental analysis.

It is obvious that many of these applications involving high power ultrasound and vessels could be optimised to varying degrees if a resource-efficient method of predicting the regions of cavitation is implemented through reliable pressure field distribution. In part, this work will attempt to demonstrate that through a combination of field characterisation techniques and advanced finite element analysis, optimisation may be implemented through a virtual prototyping of the ultrasonic system.

### 2.3.2 Power Ultrasound

A broad range of industrial applications of high power ultrasound, often referred to as power ultrasound, have been in use for over 50 years. The commercial development of ultrasonic technology in this manner has created a multi-million pound market in a variety of industries ranging from automotives to the textiles. Some of the more common applications are outlined in this section with emphasis placed on measurement and, where relevant, simulation methods practised in order to aid
development. Other notable applications of high power ultrasound that will not be expanded upon include [28]:

- Sterilisation of medical instruments
- Ultrasonic machining of brittle materials

### 2.3.2.1 Cleaning

Ultrasonic cleaning is perhaps the oldest industrial application of power ultrasound (excluding SONAR). It continues to be used in numerous industries ranging from semiconductors to engine parts due to its low cost and efficient results [29]. The main advantage of ultrasonic cleaning over traditional methods is the absence of brushes in the process, with the effects of cavitation in the load medium being the main mechanism of the cleaning procedure. This ‘brushless scrubbing’ allows ultrasonic cleaners to reach normally inaccessible places in objects with complex internal cavities that would be otherwise troublesome to clean. Furthermore, this advantage is heightened somewhat if the cleaning tank is suitably designed to generate cavitation bubbles uniformly throughout the liquid, or specifically in targeted regions. A typical cleaning bath arrangement is shown in Figure 2.3.
Ultrasonic cleaning is more effective on hard materials such as metals, glass, ceramics and plastics, which all reflect rather than transmit sound. Typical power densities utilised in most cleaning applications are relatively low compared to other high intensity operations e.g. welding. Counter-intuitively, attempts to transmit more energy into the load medium can hinder many of the beneficial aspects of the cavitational effects. Increased cavitation will be produced at the active faces causing disruption to the acoustic energy flow into the system and dramatically reducing the uniformity of the bubble density in the load, not to mention the increased damage to the transducers due to locality of collapsing bubbles. This presents an obvious need for well designed vessels to ensure the solution to inefficient cleaning operations is not to simply increase the drive power.

As mentioned in Section 2.1 the majority of high intensity applications function at relatively low frequencies and ultrasonic cleaning is no exception. Operational frequencies generally range from 20 to 50 kHz depending on the task. For example, a
25 kHz cleaner will have more cleaning prowess than a 50 kHz cleaner since the likelihood of cavitation effects is higher at lower frequencies. However, lower frequencies can prove damaging to delicate parts hence 50 kHz and above may be preferable for some applications, i.e. the semiconductor industry. In terms of health and safety, higher frequency cleaners are also quieter due to the lack of energy in the audible range [30].

Despite being an established technology for over 50 years ultrasonic cleaner design has not evolved significantly. A reason for this may be the fact that these units are very cost-effective and there has been no requirement to optimise the design practices employed. Nevertheless, the gradual deterioration of the tank due to cavitation erosions is a problem that is common to all cleaners, and one that may be alleviated to some extent from the implementation of more precise design strategies. One such strategy involves the use of finite element analysis (FEA) to predict regions of cavitation and pressure nodes for a particular arrangement. Several articles have been published in recent years that utilise a variety of simulation techniques for this purpose, most notably [31, 32, 33]. The prospect of optimising system performance and longevity through mapping cavitation zones and acoustic intensities within these vessels using a simulation approach is attractive. However, difficulties in the experimental measurement of the spatial variation of internal pressure make the verification of simulation data problematic at best.

2.3.2.2 Welding

One other major, long-established application of power ultrasonics that has successfully permeated industry is the welding of thermoplastic joins with high
intensity ultrasonic devices. The process itself progressed very quickly from the
development stage in the 1960’s to widespread use in the assembly of toys,
appliances, and industrial thermoplastic parts by the early 1970’s [29]. It is an ideal
technique for modern manufacturing; the process is fast and clean, does not need a
skilled operator, requires no consumables and lends itself readily to automation for
mass-produced parts where plastics have replaced metals and glass as the main
resource.

Plastic welding is primarily a thermal operation; the local temperature around the
target join is increased to sufficient levels to allow welding due to the mechanical
stresses generated by the high power ultrasonic equipment. However, unlike
conventional thermal techniques there is no indiscriminate heating of the surrounding
material and hence no unwanted component distortion. This advantage is partly due to
the fact that most thermoplastics exhibit favourable characteristics for ultrasonic
welding i.e. the ability to transmit and absorb acoustic energy, as well as low thermal
conductivity. In addition, since the heat is generated within the materials and
transferred via the ultrasonic tool, it is entirely possible to accomplish welds in places
that would otherwise be inaccessible to conventional welding methods.

Ultrasonic welding uses comparable frequencies to other power applications
described in this Section (~20 kHz), but differs in several ways; the functionality is
not reliant of the effects of cavitation, much higher power densities are required, e.g.
over ten times that which is used for cleaning, and the application of acoustic energy
is delivered through an ultrasonic horn. Optimisation in the development of this
technology is primarily achieved through improvement and innovation in horn design as opposed to acoustic field mapping techniques.

2.3.2.3 Cutting

The cutting of various materials, from bone to confectionary, using ultrasonic methods has advantages over conventional cutting mechanisms [34]. For example, applying normal cutting methods to soft products can result in a great deal of waste produce and imprecise performance. Performing such tasks with an ultrasonically excited blade allows highly precise cuts, very little waste and often improved process times. Furthermore, in the medical industry a great deal of interest surrounds the cutting of bone with ultrasonic saws. Conventional cutting causes problems for patients and doctors alike; unsmooth cuts, unwanted heat and bone particles imbedding themselves in neighbouring soft tissue are some of the main issues, although in other applications this heat can prove useful.

Typically ultrasonic blades are designed to resonate in the longitudinal mode of vibration in the range of 20-40kHz. However, problems with blade durability and inefficient coupling of energy in the system are present in many operations of this technology. Nonlinear modal coupling with other less desirable modes of operation and high stress conditions at certain regions of the structure compared to others are some of the main causes for blade failures and the like [35]. Indeed, these efficiency problems have a great deal of read-across when considered with other high power applications of power ultrasound.
For this reason, recent research has focussed on optimising the design of these components through extensive FE modelling and accurate vibrational and stress measurement tools [36, 37]. These are used to create virtual prototypes of blade designs which are then modified to reduce spurious mode excitation and limit regions of adversely high stress. Novel multiple blade designs have been constructed that demonstrate this premise [38], and this work has been further extended into the medical field to help develop a new generation of bone saw that will reduce vibrations and temperature increases.
2.4 High Intensity Focussed Ultrasound (HIFU)

HIFU is one application of ultrasound that is ambiguous in its definition as it can be described as both a high power and a high frequency application. The biomedical effects of high intensity ultrasound for tissue destruction have been known since the beginning of the 20th century. However, the first instances of work to consider the potential applications of HIFU were published in 1942 and subsequently built upon in the 1950’s, when tissue destruction was achieved deep within the brains of monkeys and cats. These researchers then progressed into the elementary treatment of Parkinson’s disease and other neurological conditions during the 1950’s and 60’s, although technological limitations restricted their progress [39]. More recently, interest in implementing HIFU has been resurrected due, in part, to the success of other ultrasonic techniques in the treatment of medical conditions, i.e. lithotripsy for the non-invasive treatment of kidney stones. Technological advances in transducer manufacture, beam guidance, monitoring and exposure controls have made HIFU commercially viable in comparison to competing technologies. Indeed, the use of HIFU in various cancer treatments has been widely reported for several years [40].

HIFU is basically a non-invasive surgical technique for the ablation of regions of target tissue, i.e. tumours, without adversely affecting healthy surrounding tissue. A highly focussed beam of ultrasound is utilised to concentrate acoustic energy into a focal region typically 1.5 x 15 mm in length parallel to the acoustic beam. These dimensions are very much application specific and dependant mainly on source geometry and operating frequency, which can range from 0.8 to 5 MHz. A simple depiction of a HIFU operation is shown in Figure 2.4. HIFU causes tissue damage
through two predominant mechanisms; the conversion of mechanical energy into heat via the absorption of ultrasound by the transmission tissue, and cavitation due to gases within the tissue. As cavitation is often unpredictable, especially within biological media, it was often not the objective of HIFU treatment and was viewed more as a consequence rather than a goal. Conversely, local temperature increase within the tissue is both more repeatable and more predictable, i.e. easier to simulate, than cavitation [41] and was the preferable method in the early clinical applications of HIFU. Notwithstanding, this preference appears to be diminishing as experience with HIFU grows [42]. Some of the common thermal effects associated with HIFU treatments and typical devices employed will be presented in the following sections.

Figure 2.4. A Typical HIFU device generating a focal region within biological tissue.
2.4.1 Thermal effects of HIFU

As an ultrasound wave propagates through tissue some of its energy is converted to heat due to absorption in the medium. The amount of heat generated in this manner depends on a number of factors, including: absorption co-efficient of the tissue; shape, size and active material of the transducer; input power levels; frequency content of the wave; exposure time; pulse interval and the properties of the intervening tissue between source and target. This small local change in temperature is usually a short-lived effect and the heat dissipates rapidly. However, if the region is exposed to prolonged bouts of acoustic energy the temperature equilibrium is altered and a net rise results. Providing suitably high temperatures can be achieved within the tissue (approximately > 56°C) swift thermal toxicity occurs, causing irreversible cell death through coagulative necrosis. In addition, these temperatures need not be maintained for any significant period of time to elicit the desired effect, an exposure time of less than one second has proved to be sufficient [39]. HIFU treatments are well suited to this type of application as rapid temperature increases (often in excess of 70°C) are induced within the small focal regions.

Within the focal regions in HIFU fields, the necrosis of tissue due to a sharp temperature rise forms an ellipse shaped area of damaged tissue known as a lesion. The implication being that a number of these lesions can be formed in a target tissue, such as a tumour, effectively killing the diseased cells in that area. However, there exists the possibility of damaging the nearby healthy tissue should the focal region not fall where intended. In addition, there is also the prospect that excess heat may transfer to the surrounding regions and cause unwanted necrosis. Restricting the length of the excitation signal and accurate characterisation of the HIFU devices are
some of the methods employed to reduce the risk to healthy tissue. Therefore, proper selection of the active device is essential for the safe and effective implementation of HIFU for non-invasive surgery.

2.4.2 HIFU Devices

Transducers used for HIFU applications must fulfil one basic criterion; they need to be capable of generating a focus with a desired shape some specific distance from the active face. This Section will discuss some of the devices that are suitable for such a purpose and their relative merits with regard to HIFU. For a more thorough discussion on the distinctions between pure piezoelectric ceramics and piezoceramic composites the reader is referred to Section 2.5.1.

Fundamentally, the most straightforward technique to manufacture a HIFU device with a tight focal region is to machine the radiating surface into a concave-type shape, as shown in Figure 2.4. Indeed, many rudimentary HIFU devices are fabricated in this fashion with a specific radius of curvature machined into a spherical or rectangular piezoelectric ceramic, to produce a required depth of focus at a particular frequency [39]. Obviously, this technique does not allow for much versatility in operating parameters once the device has been manufactured. The operational bandwidth can be extended slightly through the use of backing blocks and matching layers, allowing a slight deviation from the main excitation frequency in order to adjust the focal depth, although input power decreases as the drive deviates from the transducer electrical resonance frequency. Another disadvantage of machining a ceramic block for a HIFU device is the significant impedance mismatch between the device and transmission load resulting in an inefficient transfer of acoustic energy. Again, matching layer
techniques can be applied to reduce the impact of this problem. Nevertheless, this leaves the medical technician with a device effective in producing an intense focus but with limited application diversity due to its mechanical rigidity.

Piezoceramic composites offer several advantages over monolithic ceramic blocks and these advantages can be harnessed with regard to HIFU devices. Superior acoustic impedance characteristics when transmitting into tissue or water (which have approximately the same acoustic impedance), increased transmission efficiency and the possibility of creating arrays make piezoceramic composites an attractive alternative to simple machined ceramic transducers. Configuring the transducer in an array format enables each element of the composite to be individually addressed for both transmission and reception purposes. By correctly timing the firing signals into each element a virtual concave surface can be formed, producing a beam with the desired focal range [43]. Moreover, it is possible to electronically steer the beam in all spatial directions through a 2D array configuration, without recourse to mechanical movement in these planes. However, this type of arrangement is more common in biomedical diagnostic applications. Piezoceramic composites have recently been used in annular phased array HIFU devices, with good results reported [44].

Despite the flexibility and increased number of options afforded by incorporating phased-array technology in HIFU treatments, delivering large amounts of power to a relatively distant target area (> 80 mm from device) via a single device presents potential safety concerns. A larger aperture is required and although the majority of energy resides in the focal region, areas of healthy tissue between the source and target may be unintentionally damaged. Furthermore, at higher intensities non-linear
and cavitation effects become more prominent, altering the predicted profile of the acoustic beam. Other possible failings include near field heating produced when scanning a large area due to beam overlap, and off-focal heating due to the presence of side lobes. One approach to solving this problem is to utilise the overall power generated by the superposition of several smaller low power devices located at suitable angles [45]. The proposition being that individual, low power beams will not damage healthy tissue outside the combined focal region.

It is evident that accurate characterisation of the field profiles generated by HIFU transducers, is one of the major challenges in ensuring HIFU treatments are safe and effective non-invasive surgical techniques. This is a unique problem as HIFU utilises both relatively high frequencies and high intensities, and hence, the field measurement difficulties associated with both must be tackled simultaneously. Unfortunately, many measurement techniques are proficient at one or the other, but rarely both. A non-invasive method with a large bandwidth and potential for high spatial resolution has the potential to resolve the measurement limitations underlined.
2.5 Measurement Methods

Most recent research regarding process-enhancement optimisation-techniques has centred on the prediction and creation of high pressure regions within a test cell. Here, favourable conditions for transient cavitation can be optimised through specific cell design and transducer arrangement. Indeed, reasonable correlation has been achieved between theoretical projections and experimental determination of prolific cavitational zones [31-33]. However, the accurate quantification of the fields within these cells has not generated a significant level of interest and subsequently there exists a lack of published material on the subject.

High power ultrasonic fields can be extremely difficult to characterise, often due to the cavitational activities themselves; not only can cavitational effects cause damage to any measurement instrumentation being used, but regions of dense bubble populations can also scatter the source acoustical signal under investigation. This often facilitates measurements being obtained under non-cavitational conditions; several methods to suppress or limit cavitation are outlined in Section 2.2.3. Nevertheless, conducting measurements in non-cavitating fields may not yield true pressure distribution experienced during a high power application, but it will identify locations where cavitational sites are likely to occur when sufficient power levels are reached.

Traditionally, hydrophones are the principal device for field characterisation with use in medical ultrasound for exposure quantification widely reported. There are a number of important factors to consider in hydrophone design whether it is
piezoelectric ceramic based or, more recently, piezoelectric polyvinylidene fluoride (PVDF) membrane. The device itself should be non-perturbing to the acoustic field in order to minimise any detrimental effect on the field profile, although the physical nature of the probe makes complete non-invasive measurement impossible in reverberant environments. Furthermore, many hydrophones suffer from a lack of uniform response over a wide range of frequencies while still maintaining sensitivity, particularly below 200 kHz where the majority of high power applications operate. In addition, any measurement probe must be robust enough to withstand the hostile fields associated with high power ultrasound measurement, where PVDF membrane devices in particular are very susceptible to damage. Notwithstanding, hydrophone devices by their very nature generally can be quite delicate and fragile as designs strive to attain increased levels of sensitivity, spatial resolution and non-invasiveness, making damage to the active element is the main concern. This may explain the lack of literature available on the use of hydrophones for the measurement of acoustic fields generated by high power applications, with the exception of lithotripsy in the medical field where apprehensions over safety has facilitated the development of robust probes, at the expense of sensitivity [46]. This has lead to the development of the more durable optical fibre hydrophone that demonstrates marked improvements in both spatial resolution and sensitivity over conventional counterparts [47].

Nonetheless, these devices still require insertion into the load medium causing a direct impact on the pressure fields present and on subsequent measurements. Manipulation of probe position within a sealed container is also very problematic and is not conducive to obtaining accurate field profiles. This difficulty in taking precise measurements leads to subsequent problems in validating any simulation data that
may be available for comparison. Given the need for reliable simulation data, the Finite Element Analysis (FEA) code PZFlex [48], which offers the ability to fully simulate the ultrasonic field generated in sealed vessels for a variety of systems and devices, will be employed for this purpose and is discussed in Chapter 4. In this Section, the various quantitative techniques briefly mentioned will be discussed in more detail with particular emphasis on their applicability to high power field measurement. An excellent alternative review of measurement methods for high power ultrasound can be found in [49].

It should be noted that other, less popular techniques will not be covered in this Section. Some of the more common ones are listed for the readers’ interest: radiation pressure balances [4], holography [50], chemical detection [8], and thermal measurements [30]. Of these, thermal methods have been used with some success in high power field measurements to provide a simple means of evaluating the power in an ultrasonic cleaning bath.

### 2.5.1 Piezoelectric Ceramic Hydrophones

Piezoelectric hydrophones are detectors based on transducers which respond directly to pressure variations in a load according to the direct piezoelectric effect [51, 52]. The Curies discovered that a mechanical deformation applied to a quartz crystal resulted in an electric charge being produced on the surface, where, in terms of acoustic measurement, the mechanical deformation is the acoustic disturbance. Later, the inverse piezoelectric effect was discovered [53], in which if a piece of quartz is subjected to an electric field across it then a mechanical deformation will occur. Since the 1950’s, modern piezoelectric materials exhibiting more advantageous
characteristics for the transmission and reception of acoustic signals have superseded quartz as the active material. The most common synthetic piezoelectric ceramics are based upon lead zirconate titanate (PZT), although others such as lead metaniobate and modified lead titanate are frequently used. For practical measurement, the charge generated within the material due to an incident mechanical wave is detected by electrodes deposited on opposite faces of the piezoelectric element. The subsequent electrical signal can then be amplified as a voltage to produce a visual representation of the acoustic waveform on an oscilloscope. The temporal signal can be processed as required to obtain additional information such as spectral content. However, accurate depiction of the acoustic field is by no means as trivial as the detection method infers. Early piezoelectric ceramic hydrophones are summarised in [4].

A conventional piezoelectric ceramic hydrophone schematic is shown in Figure 2.5. It comprises of an active piezoelectric element, a backing block for damping and a matching layer. The matching layer and the backing block serve to optimise the characteristics of the probe by both widening bandwidth and increasing sensitivity [54], although there exists a permanent trade-off between the two. There are a number of important factors to consider in hydrophone design, particularly the piezoelectric ceramic variant. Firstly, the device should be relatively non-perturbing; this becomes more of an issue at higher frequencies due to the corresponding decrease in acoustic wavelength. Nevertheless, for some lower frequency applications it would not be desirable to disrupt the standing wave patterns integral to the process through the introduction of a measurement probe. Secondly, the device should also have a uniform frequency response over the bandwidth under scrutiny, while maintaining reasonable sensitivity. Again, this can prove more of a problem in characterising high
frequency (> 3 MHz) medical diagnostic equipment, or in the measurement of very short transient signals associated with the pulsed operation of such devices. This will not be the case in the realm of power ultrasonics. However, sensitivity at frequencies in the region of 20 kHz is often well outside the normal 3dB range of many typical hydrophone probes [55] and custom designed probes must be considered in these instances.

Piezoelectric ceramic composites (commonly known as piezocomposites) address many of the disadvantages associated with monolithic piezoelectric ceramics i.e. high acoustic impedance, limited bandwidth, spurious modes dependent on physical geometry, while retaining the advantages of high electromechanical coupling and high permittivity. A discussion on these devices will follow.

Figure 2.5. Arrangement for a typical piezoelectric probe.
2.5.1.1 Piezoelectric Composites

Piezocomposites typically comprise of two constitute phases; the active piezoelectric ceramic, and the passive polymer phase. Through varying the proportions of each in one device (ceramic volume fraction) it is possible to produce a transducer that can more efficiently transmit acoustic energy into a load medium than a monolithic piezoelectric ceramic alone. Alterations in ceramic volume fraction and polymer characteristics create a device possessing a lower acoustic impedance and a higher electromechanical coupling coefficient than that of a pure piezoelectric ceramic. Similarly, the increased bandwidth due to favourable acoustic impedance matching conditions to the load medium and the absence of lateral modes make piezocomposites a preferential choice for many hydrophone applications, particularly at lower frequencies. Indeed, coupled with matching layer and backing block technologies [54] piezocomposites can be easily fabricated for a range of specific tasks through the manipulation of device parameters such as volume fraction, pillar dimensions and constituent materials.

Smith’s excellent review of piezocomposites in ultrasonic transducers [56] describes the various microstructure arrangements (known as the connectivity of the device) that may feature as the active element in an ultrasonic transducer. The term connectivity refers to the orientation of the ceramic within the polymer and the number of orthogonal directions in which each phase is continuous. For example, in a typical 1-3 the ceramic is in the form of elongated pillars, known as rods, connected continuously in one direction, whereas the polymer that surrounds the rods are connected continuously in 3 directions. This configuration is explained pictorially in Figure 2.6. These simple geometries can be manufactured by a dicing process where
two sets of parallel grooves are cut orthogonally into the monolith and subsequently filled with an epoxy resin. A more thorough account of connectivity and the manufacturing methods employed to produce piezocomposites can be found in [54] and [57]. Of the designs available, it has been established that the 1-3 connectivity is the most productive arrangement in many ways for the majority of applications [58].

Figure 2.6. Arrangement of a typical piezocomposite probe in 1-3 connectivity.

Nevertheless, it should be noted that piezocomposite hydrophones will be subject to the same shortcomings as monolithic probes when used for the characterisation of high power fields, i.e. perturbation of the field and potential damage due to cavitation. The advent of piezoelectric polymers offered an alternative, and potentially superior, technology for quantifying acoustic pressures. Details of these will now be presented.

2.5.2 Polyvinylidene Fluoride (PVDF) Devices

While ceramic and piezocomposite hydrophones are adequate for characterising CW or narrow band tone burst fields used in therapeutic applications, the dimensions of the active piezoelectric element and probe housing make them intrinsically multi-
modal and unsuitable for measuring broadband diagnostic pulses. Nevertheless, the discovery of piezoelectricity in the polymer PVDF by the Japanese in 1969 provided the potential for pressure sensors without the problems associated with the ceramic devices. The main advantages of using PVDF as a sensor over ceramic are; a much better acoustic impedance match to water and tissue, its availability in thin flexible sheets and a linearly broadband, flat frequency response. Admittedly, PVDF devices are primarily geared for use in the medical field and this has indeed been the cause of their emergence over the years with PVDF now the established ‘gold standard’ sensing material for hydrophone based measurements in the biomedical industry. There is potential to use such devices in a high power environment under suitable circumstances i.e. avoiding contact with regions of cavitation. PVDF hydrophones are typically categorised as membrane devices, however, PVDF can also feature as the active element in needle-type devices, in addition to piezoelectric ceramic with the former now being more common.

2.5.2.1 Membrane Devices

As PVDF film is available in large, thin (approximately 5 to 110µm thick), flexible sheets of similar acoustic impedance to water, it is a natural choice for designing hydrophones for acoustic field measurement in biomedicine and SONAR. The most common design for such a device comprises of a large sheet of PVDF with gold or chromium electrodes vacuum deposited on the surface, stretched across an annular frame, also known as the hoop-supported membrane approach. Metal film leads are evaporated onto both sides of the membrane and the small overlap formed determines the active area of the device. Using this technique a relatively small active area (approximately 40µm) can be produced on such a membrane, shown in Figure 2.7. A
detailed description of PVDF membrane hydrophone manufacture, calibration, operation and simulation is available in [59]. Although the techniques described in [59] are used in the characterisation of air coupled devices, they are based on established methods for a water load.

![Membrane hydrophone arrangement](image)

Figure 2.7. Membrane hydrophone arrangement

Given that characteristic acoustic impedance of PVDF is well matched to that of water and assuming the membrane is thin compared to the acoustic wavelength, which will nearly always be the case, membrane hydrophones have the advantage of causing minimal disturbance to the acoustic field under investigation. The ultrasonic beam does not ‘see’ the PVDF although the complete device diameter may be greater than the acoustic wavelength, contrary to ceramic probes dimensions that are often required to be less than the acoustic wavelength. Hence, membrane hydrophones negate some of the complications associated with their ceramic counterparts, such as...
the presence of frequency dependent modes due to the active element dimensions and probe structure.

Membrane hydrophones find their use almost exclusively in the characterisation of medical fields in water below the cavitation threshold, with only a few exceptions [60], and are often used in degassed water to reduce the risk of cavitation occurring. When operating in cavitating fields the device may suffer from localised damage to the PVDF membrane due to bubble collapse and degradation of the electrodes. This can adversely affect sensor sensitivity and signal reproducibility with continued use. Attempts to prevent this from occurring typically result in devices that no longer accurately represent the acoustic waveforms under investigation. Moreover, the fundamental radial mode frequency of the PVDF film is related to the diameter of the device membrane rather than the active area alone and therefore, at lower frequencies the response of the device is no longer flat. This may cause difficulties when attempting to characterise the low frequency fields generated in the majority of high power applications, even under the assumption that cavitation within the load has been minimised.

2.5.2.2 Needle Devices

Needle-type hydrophones generally consist of an active element approximately 0.5 mm in diameter, more commonly PVDF film but occasionally piezoelectric ceramic based, mounted onto the end of a hollow cylindrical tube with an outside diameter close to that of the active element. The cylinder is filled with an acoustically absorbing material (backing) with an acoustic impedance much greater than that of the membrane. The outer surface of the cylinder is connected electrically to the film
surface and the inner surface is attached to an insulated wire placed inside the tubing. Figure 2.8 depicts the arrangement of a typical needle hydrophone.

![Diagram of needle hydrophone arrangement](image)

The design of needle-type hydrophones incorporating PVDF as the active element is very similar to the ceramic based designs, but with exceptionally contrasting frequency responses. Indeed, ceramic needle hydrophones can experience unpredictable structure in both their directional and frequency responses due to radial resonance modes, reflections and mode conversions in the active element and backing material. Furthermore, the relatively high acoustic impedance of the active element, when compared to water, causes a distinct perturbation of the field being measured. The geometry of the needle-type probe has afforded it some unique advantages over its membrane and ceramic counterparts; it is easily adaptable for measurements in confined spaces such as *in vivo*; in situations involving measurements near the source or under CW excitation, where membrane probes may generate unwanted reverberations, the needle hydrophone provides a cleaner signal with less perturbation of the field, and for certain transducer geometries within an enclosed environment, the
needle hydrophone has better access than other types of devices. The difference between a membrane and needle hydrophone placed in a CW field is investigated in [61] with the needle probe demonstrating less disturbance to the harmonic field. Another notable advantage of the needle hydrophone is that a well designed probe can exhibit a directivity pattern that is close to an ideal piston. These devices are also fairly robust, making them viable suitors for the transition from medical applications to the characterisation of high power fields in a lower frequency regime. However, the presence of radial modes at lower frequencies due to membrane geometry; deterioration of the contact between the wire and active element over time and a roll-off in the low frequency response caused by diffraction at the tip are concerns for low frequency measurements. An excellent review of PVDF’s influence on medical ultrasound field techniques and standards can be found in Harris’ comprehensive article [62].

### 2.5.3 Fibre-Optic Devices

The proposition of using light to detect and quantify an acoustic field is not a new one. Extensive literature exists dating back to the 1930’s with the initial observations of Debye and Sears, and Lucas and Biquard on the diffraction of light by an ultrasonic field [63] lead to the start of a field known as acousto-optics. It could even be said that the first interaction between sound and light pre-dates the previously mentioned publication as flames where used in the 19th century as a qualitative measurement of an acoustic wave [64]. The field of acousto-optics and related techniques will be discussed in greater depth later in this work (Chapter 4). This section is focussed on the use of probes incorporating optics for the characterisation of ultrasonic fields for a range of applications, while emphasising their potential for high power measurement.
Field measurements by fibre-optic devices have the potential to overcome some of the problems associated with conventional piezoelectric or PVDF hydrophones. The degree of perturbation caused by conventional devices is always a cause for concern in that the overall effect on the field dynamics can be hard to ascertain. In addition, manufacturing difficulties in producing active elements small enough to minimise these effects, while maintaining a broadband response and a desired level of sensitivity, have led researchers to explore possible optical detection methods to address these limitations. Optical methods can offer the following; minimal intrusion of the field, reduced element size of several microns limited only by the diameter of the optical fibre, near omni-directional response, linear broadband frequency response, relative manufacturing ease and a degree of ruggedness not often associated with the conventional probes. Furthermore, an optical sensor known as a laseroptic hydrophone [65] has been shown to provide information on cavitation occurring near the active element. This device merely consists of light from a laser diode coupled into a glass optical fibre, of 50μm diameter, with the end placed into the load medium. A photodiode is utilised to detect the light reflected back along the core from the glass/water boundary. Assuming an acoustic wave is incident upon the end of the fibre, the density and hence refractive index of the water at the fibre will be modified in proportion to the compression phase of the wave, therefore modulating the amount of light reflected at the boundary. The sharp discontinuity in refractive index caused by the presence of an air bubble created by cavitation is easily detected by the sensor. A similar device for use in the measurement of shockwaves displays a similar resistance to bubble collapse near the active element [66].
There are two types of optical sensor that are generally used for ultrasonic field characterisation; probe based systems that rely on a physical change to detect pressure, i.e. deformation of a surface; and non-invasive systems that are based on the diffraction of light by an acoustic field. The former variety is known as a fibreoptic hydrophone and demonstrates potential in both conventional and high power field characterisation [47, 67]. Of this variety there are there are two designs of merit; the laseroptic probe discussed previously [65, 66], and a more subtle design featuring a polymer film at the end of an optical fibre. The device itself consists of 25µm thick polymer film deposited onto the end of a single mode optical fibre, diameter 6µm. Two aluminium mirrors are evaporated onto the fibre end and the polymer with reflective coefficients of 8% and 70%, respectively. The active area of the probe is approximately equal to the optical fibre diameter and all other important dimensions can be found in Figure 2.9. The detection mechanism is based upon the acoustically induced changes in the optical thickness of the polymer film acting as an interferometer. Significantly, this device demonstrates sensitivity levels comparable to a much larger PVDF membrane device (0.2mm$^2$ active area), while offering lower directional sensitivity than that of a PVDF needle device (0.075mm$^2$ active area). Due to the ease of manufacture for such a probe, disposable sensor heads could be developed to make characterising high power fields economically viable in terms of potential damage to the sensors inserted into these hostile environments.
Figure 2.9. Arrangement for a fibre optic polymer film hydrophone.

The other variety of fibre optic sensor is based on the diffraction of light by ultrasound. These probes are essentially non-invasive as they do not interact with the acoustic beam, unless used in a reverberant environment. Functionality of these sensors is based on Raman-Nath light diffraction [68], which states that when a beam of light passes through an acoustic field, diffraction of the light beam takes place and by measuring the amplitude and frequency of the detected beam, information about the acoustic field can be extracted. In essence, the acoustic field acts as a diffraction grating. This concept will be expanded upon later in Chapter 3. In contrast to other ultrasonic hydrophones that require to be placed in the acoustic field, this technique requires no physical interaction with the acoustic beam, and hence, does not perturb the field [69]. The transmitting optical fibre is placed in a water tank perpendicular to the acoustic beam axis and on the same plane as the focal region of the transducer. The receiving fibre is placed directly opposite with a gap of approximately 15 mm separating the two. The detected diffraction patterns are coupled into an avalanche photodiode and then electrical signal displayed on an oscilloscope, as shown in a
simple schematic in Figure 2.10. Sensitivity for this technique is reported as being lower than traditional PVDF methods; however, this is offset by the potential advantages gained from a more uniform directional response and increased spatial resolution. Nevertheless, this technique may not be applicable to reverberating high power fields as both optical fibres may incur damage due to bubble collapse and erroneous measurements may result as the rest of the optical fibre in the system interacts with the ultrasonic field.

Figure 2.10. Typical experimental arrangement for Raman-Nath fibre optic sensor.
2.5.4 Non-Invasive Techniques

The potential to characterise an acoustic field without the insertion of a sensor is an attractive proposition. Removing the perturbation caused by a hydrophone, regardless of how minimal, can provide data that represents a more accurate representation of the pressure profile. Several methods exist that are able to provide information about the ultrasonic field without physical interaction. Generally, they can be split into qualitative and quantitative techniques and, intuitively, the most effective ones in both categories are optical based technologies.

2.5.4.1 Optical Diffraction Tomography (ODT)

Optical diffraction tomography (ODT) combines the diffraction of light by an acoustic beam and tomographic routines to form images of pressure in a chosen plane perpendicular to the acoustic axis. An ultrasonic transducer with four degrees of freedom (x, y, z and rotational) is set up in a water filled tank, through which a beam of monochromatic laser light, typically from a Helium-Neon source, is transmitted. The diffracted light signal exiting the tank is received by a photodetector, where the demodulated intensity is proportional to the average pressure through the width of the field. By taking a series of parallel measurements as described and then rotating the transducer in monotonic angles through 180 degrees, simple tomographic reconstruction algorithms can then be implemented to create an image of pressure at an arbitrary distance from the source [70, 71, 72, 73, 74]. This technique is completely non-invasive and has the potential for obtaining greater spatial resolution than hydrophone methods. Moreover, the absence of a detector in the field enables
accurate measurement of the acoustic nearfield without the presence of unwanted reflections.

2.5.4.2 Interferometry

Laser interferometry has been employed in two distinct ways for ultrasonic field characterisation. The first manner is similar to ODT as tomographic scanning routines are used to generate images of pressure, but different in that the phase modulation due to the light traversing the acoustic disturbance twice is used to evaluate pressure. This technique will be described in the detail in Chapter 3. The other method is based on quantifying the displacement of a membrane caused by an acoustic field, and will be described here.

It would be inaccurate to describe this second arrangement as a pure non-invasive field characterisation technique as a thin plastic reflective membrane (known as a pellicle) is placed in the path of the acoustic beam. One surface of the pellicle reflects the optical beam of a laser interferometer, which is used to determine the absolute displacement of the membrane and hence of the acoustic field. From this displacement measurement, the absolute acoustic pressure can be calculated. As the influence of the pellicle (of similar dimensions to a PVDF hydrophone) on the field is minimal and, as such, this method is categorised with other non-invasive techniques for the purposes of this review.

Primarily utilised as a reliable means of calibrating hydrophones [75], this technique was later adapted for pulsed field characterisation [76], and then expanded upon by incorporating a 3D mechanical scanning system for transducer movement to facilitate
complete field mapping, if desired [77]. With regard to high power field measurement, damaging the fragile membrane would pose the most significant problem.

2.5.4.3 Schlieren

Schlieren imaging of ultrasonic waves is traditionally a qualitative technique that has proven useful for the visualisation of acoustic beams incident upon and reflected from various surfaces. The basic theory behind a schlieren system is from Raman and Nath’s treatment on the diffraction of light by sound [68], in that a propagating sound wave induces a change in the refractive index of the transmission medium, causing the sound wave to behave like a diffraction grating. The intensity of the subsequent diffraction pattern is proportional to the integral of pressure along the light path. Therefore, the intense pressures in the beam are represented as greater intensities in the optical signal received as the light passes through the field. A simple schlieren system is shown in Figure 2.11 (a). This can be utilised to produce striking images of acoustic fields under free field conditions and reflecting from a surface.

Since schlieren visualisation requires a transparent medium for operation the obvious limitation is it must be used in conjunction with a non-opaque load fluid. Furthermore, it does not accurately represent the pressure field as it forms a 2D measurement from a 3D sample.
2.5.4.4 Laser Vibrometry

This method is a novel utilisation of laser scanning vibrometry, based on interferometric principles, that has been applied with great success in the measurement of the vibrational displacement in a variety of transducers [78, 79]. By placing the transducer in a tank with the acoustic axis perpendicular to the laser light and securing retro-reflective material to the other side of the tank, it is possible to measure the average change in refractive index through the width of the beam. In this manner, a complete scan of the average intensity of the acoustic beam can be generated [80]. Providing gated excitation is used and reverberations minimised, this technique can provide a reasonably accurate spatial representation of the acoustic intensity distribution from an ultrasonic device. However, the refraction of the incident laser light by the tank walls may introduce some measurement errors if not accounted for. This methodology also suffers from the same difficulties as

Figure 2.11. (a) Experimental arrangement for a conventional Schlieren system.
conventional schlieren imaging as a 2D image is formed from a 3D data set, though tomographic techniques could be employed to remedy this.

### 2.5.5 Cavitation Monitoring Techniques

Throughout this Chapter, the effects of acoustic cavitation have been emphasised as the main mechanism in many applications of power ultrasonics. For example, the high temperatures and pressures generated during inertial cavitation are essential catalysts in many of the reactions used in sonochemical applications. Although it is not the purpose of this work to present a novel means of monitoring cavitation, rather, in part it is to optimise the design of these systems that rely on cavitation through improved measurement techniques and virtual prototyping tools; a brief summary of the current methodologies for detection is warranted.

Until recently, the measurement of cavitation has been a particularly troublesome problem. Some of the difficulties in measuring cavitation have been highlighted throughout this chapter, difficulties such as; damage to sensing equipment, large transient signals, hostile environmental conditions, unpredictability, and difficulty distinguishing between the two types of cavitation. Common measurement methods include [81]; broadband acoustic emission, aluminium foil erosion, chemical effect monitoring (chemiluminescence) and sonoluminescence. Despite the attractiveness of these two luminescence techniques and their potential for high spatial resolution, the requirement for blackout conditions in optically transparent media renders them complex to implement in practice. Conversely, passive acoustic methods incur none of the complications associated with the optical techniques and, consequently, are more widely applicable.
Cavitating bubbles behave as acoustic sources when stimulated by an external acoustic field via modes generated by the bubble’s non-linear motion, emitting harmonics and sub-harmonics of the acoustic drive frequency. Hydrophones positioned inside, or fixed on the outside, of the container are able to pick up the acoustic signatures of the bubble dynamics. The amplitude, phase and frequency information of these signals can provide data on the scale and nature of bubble activity. However, obtaining spatial knowledge of the cavitating bubbles for a particular volume of liquid is difficult with conventional acoustic monitoring. This prompted Zeqiri et al [82, 83] to identify the attributes desired for a novel cavitation monitoring sensor and develop the device accordingly. The sensor consists of a thin layer of piezoelectric polymer film attached to the inner surface of a hollow, open-ended cylinder, providing measurement bandwidth from 0 to 10 MHz. The outer surface of the cylinder is coated with a specially developed cavitation shield material that is highly attenuating to acoustic signals at megahertz frequencies. This provides the sensor with a degree of spatial resolution as any acoustic signals characteristic of acoustic cavitation arise from events occurring within the cylinder volume. Moreover, the coating material has an acoustic impedance similar to water therefore ensuring the sensor is minimally perturbing to the field under investigation. Furthermore, it is possible to increase spatial resolution by reducing the internal diameter of the sensor.
2.6 Summary

This chapter has outlined the evolution of power ultrasonics as a branch of acoustics and provided brief descriptions of its use in ultrasonic cleaning, welding, cutting and sonochemistry. The latter is of particular interest as it is only recently becoming viable on an industrial scale. The distinction between power ultrasonics and other disciplines has been noted and applications that are not exclusive to only one field have been highlighted, e.g. high intensity focussed ultrasound is both high frequency and power. Furthermore, the primary mechanism behind many of these applications, acoustic cavitation, has been identified and intuitively discussed with specific emphasis on the parameters that influence it and how to manipulate them for optimisation, or minimisation, purposes. In addition, some cavitation monitoring techniques have been mentioned for the readers’ interest.

A review of contemporary measurement methods for high power ultrasound with an evaluation of their respective feasibility for use in high power fields is presented. The main difficulties are; perturbation of the acoustic field, potential damage to the sensor and wideband sensitivity issues. These problems present a challenge when attempting to validate simulation data associated with many high power ultrasonic systems, and can account for the lack of design optimisation processes applied to these systems. This has lead to the conclusion that a non-invasive optical based measurement technique has the potential to mitigate many of the disadvantages associated with conventional probes, and provide a platform to accurately verify simulation data.
In the next chapter the design of a non-invasive measurement technique will be presented and assessed in comparison to established methods for ultrasonic field characterisation.
CHAPTER 3

3. DEVELOPMENT OF A NON-INVASIVE FIELD MEASUREMENT TECHNIQUE.
3.1 Introduction

Due to the limitations of many conventional ultrasonic field measurement techniques involving probe insertion into the load medium, an accurate non-invasive technique is an attractive proposition. The removal of hydrophone presence in the load medium leads to the possibility of creating more precise pressure maps and hence, a better assessment of a transducers suitability to a particular task. Given this, the interaction between sound and light provides a unique means to quantify an acoustic field in a transparent medium without altering the system dynamics during the measurement procedure.

Although the idea of visualisation of sound by light is not a new one, combining it with tomographic techniques is relatively recent. Molkenstruck and Reibold [70, 71] demonstrate that through Light Diffraction Tomography (also known as optical diffraction tomography) profiles of selected cross sections of an acoustic field, in a non-gaseous medium in the low MHz region, can be constructed through computer implemented algorithms that have their basis in x-ray tomography. Amplitude and phase profiles can be reconstructed independently through specifically chosen algorithms applied to each data set. Like many others since, the authors proceed to compare these profiles with results taken using a hydrophone probe, with the relative difference in resolution of the two methods becoming apparent. These two papers comprise the basis for future research concerning optical diffraction tomography and are consistently cited.
More recently however, the technique of optical diffraction tomography (ODT) has been applied to airborne ultrasound with Anders Holm and Hans Persson’s [72, 73] publications. The authors utilise the measured light intensity of the diffracted beam and standard tomographic techniques, akin to Molkenstruck et al’s procedure, to produce cross sectional images of an airborne ultrasonic field. R. Eriksson [74] et al have also taken this method and adapted it for the study of ultrasonic near-fields in the 10MHz range that, ordinarily, would be subject to several undesirable aspects associated with hydrophone use, with lack of resolution being the primary concern. The authors compare the results from the ODT method with theoretical beam predictions based on an ideal piston transducer and hydrophone measurements. The outcome verifies that the optical technique demonstrates increased accuracy, spatial resolution and sensitivity over the traditional hydrophone approach.

The use of interferometry, or more accurately heterodyne interferometry, for the purpose of quantifying ultrasonic pressures in a medium was investigated by X. Jia et al [84, 85, 86]. In their publications the authors describe how a propagating acoustic pressure wave has a subsequent effect on the refractive index of a medium and as a result, an effect on the phase of laser light traversing this medium. This relationship between acoustic pressure and phase variation is used by the authors to produce a waveform of ultrasonic pressure, generated by a pulsed source, at various distances from the transducer face. This technique removes the need for the relatively long and complex optical path required for high spatial resolution that is characteristic of the ODT method. One aspect of note that is commented upon by the authors is the very high sensitivity demonstrated by this technique. However, although no mention is
made of mapping the field produced, the potential of this method to produce an image of pressure is evident.

The combination of tomography and interferometry was first proposed by the French group consisting of O. Bou Matar and co-workers [87]. The authors investigate the use of optical heterodyne detection for the purpose of characterising airborne electrostatic transducer sensitivity i.e. Pascal’s per applied volt, operating between 200 kHz and several MHz. In their later work [88, 89] they continue their use of this technique into the field of mapping acoustic pressures producing three-dimensional temporal and spatial images of pressure created by piezo-composite transducers.

This Chapter will describe the fundamentals of the acousto-optic effect, principles of laser interferometry and how a combination of these two techniques can be used to quantify acoustic pressure, evaluate radiated pressure field profiles, verify tomographic algorithms. Finally, a modified scanning routine for inspection of pressure fields within vessels will be described.
3.2 The Acousto-Optic Effect

Sound waves can modulate light in amplitude and phase, deflect it, focus it, or shift its frequency [90]. It is this modulation of phase when light encounters a field of sound which provides the foundation for this investigation into the visualisation of acoustic fields. Over the years, and through the publication of a large amount of material on sound’s unique influence on light, this interaction between the two has come to be known as the field of Acousto-Optics. The diffraction of light by sound, often called Brillouin scattering after the scientist who discovered it, has been the focus of extensive research since the observation of diffracted light due to high frequency sound by Debye and Sears [91] and Lucas and Biquard in the 1930’s, prompted Raman and Nath to publish a series of papers [92] outlining a potential theory behind this phenomenon. These papers still influence present day theoretical and experimental work concerning the way in which acoustic fields affect light. With the advent of the laser in the 1960’s, precise confirmation of the effects observed by Debye and others was achieved. As a consequence of the explosion in demand for laser technology in the years that followed its creation, the number of possible applications for its use encapsulated within the field of acousto-optics broadened significantly. Robert Adler documented this observation in his 1967 paper on the interaction between sound and light [93]. Since then a multitude of innovations incorporating acousto-optics have been developed; from the ultrasonic microscope developed by Adrian Korpel, ink distribution in laser printers, to the laser television projection system also developed by Korpel.
The field of signal processing has also benefited considerably from the development of systems involving lasers and that utilise the properties of the acousto-optic effect.

With the introduction of the laser came the development of the coherent heterodyne detection techniques that form the basis of the heterodyne interferometric devices discussed later in Section 3.3. A great deal of literature surrounds this and other signal processing techniques with Acousto-Optic Signal Processing by Berg and Pellegrino [94] recommended as a thorough introduction. In addition to those applications mentioned previously, the acousto-optic phenomenon has been used more recently by various researchers to visualise sound fields and characterise various types of acoustic radiators, as outlined in Section 3.1, and is the focus of this Chapter.

### 3.2.1 Theoretical considerations

Certain aspects of acousto-optics require expansion, beginning with a basic propagating sound wave. A harmonic sound wave can be described as the longitudinal displacement, $u(z,t)$, of a medium’s particle along an axis that is the same as the direction of the wave, and is represented by the elementary equation of a harmonic wave shown in Equation 3.1

$$u(z,t) = A \sin \left[ \omega \left( t - \frac{z}{c} \right) + \phi \right]$$

where

- $A$ = Amplitude
- $\omega$ = is the angular frequency, $2\pi f$
- $\phi$ = phase angle at $x = t = 0$
- $c$ = phase velocity in the load medium
As the wave propagates through the medium a series of compressions and rarefactions are imposed at distinct regions along the wave’s structure i.e. peaks correspond to regions (or temporal zones) of compression, and troughs to dilation. Figure 3.1 illustrates this simple concept. The compressional phase results in a net pressure increase on the ambient pressure in the medium, whereas the rarefractional phase results in a net pressure deficit.

![Figure 3.1. The Effect of a propagating longitudinal sound wave on the structural lattice in the load medium with respect to a harmonic wave.](image)

### 3.2.1.1 The Piezo-Optic Co-efficient

A medium perturbed by an acoustic wave such as the one described in Equation 3.1 will be subject to changes in its permittivity corresponding to changes in the number of molecules per unit volume. Concurrently, the number of molecules per unit volume is altered when the density of the medium is altered, in this case dynamically by an oscillating sound wave. It is known that refractive index is a function of permittivity hence refractive index is also a function of density. Greater density means a greater
number of molecules in the same volume, which implies an increase in permittivity and refractive index.

\[
n = \sqrt{\varepsilon_R}
\]

where \( n \) = refractive index of medium
\( \varepsilon_R \) = relative permittivity of medium.

The relative permittivity can be found by

\[
\varepsilon_R = \frac{\varepsilon}{\varepsilon_0}
\]

where \( \varepsilon \) = the absolute permittivity of the medium
\( \varepsilon_0 \) = the permittivity of free space \( \approx 8.85 \times 10^{-12} \text{ F/m} \).

Intuitively, the increase in density is due to an increase in pressure locally caused by a propagating acoustic wave. Therefore, in a transparent medium, it is reasonable to state that changes in pressure will result in modulations to the medium’s refractive index. This relationship is considered linear under a small signal model and can be represented by a simple co-efficient

\[
\mu_{op} = \frac{\delta n}{\delta p}
\]

where \( \delta n \) = variation in ambient refractive index
\( \delta p \) = variation in ambient pressure.
\( \mu_{op} = 2.7 \times 10^{-9} \text{ Pa}^{-1} \) in air.

The refractive index of a region with a disturbance present can be written fully as
\[ n = n_0 + \delta n \]  

where \( n_0 \) = the refractive index in the undisturbed medium.

If the change in refractive index is caused by a harmonic sound wave, it can be more accurately represented as

\[
\delta n(t) = n_1 \sin \left[ \omega \left( t - \frac{z}{c} \right) \right]
\]

where \( n_1 \) = the maximum change in refractive index

\( c \) = the speed of sound in the medium.

This dynamic change in the ambient refractive index of a transparent medium, and hence the density of the medium, has certain implications on a beam of light traversing the ultrasonic disturbance. The regions where the volume of molecules is greater will alter the speed at which the light will travel through them; hence, a delay is introduced. Raman and Nath [68] state that if the refractive index of a medium is the same for all points a beam of monochromatic light with plane phase incident upon the medium will emerge unchanged. However, if a disturbance is present within the medium (e.g. by the presence of an oscillating sound field) the incident light will emerge from the medium with variations in phase corresponding to the refractive index at different parts of the medium. Succinctly, changes in the refractive index will produce changes in the phase of a beam of light perpendicular to the disturbance as illustrated in Figure 3.2.
Essentially, the modulation of a non-opaque medium’s refractive index induces a phase delay on a light beam traversing the acoustic field. Quantifying this phase delay provides a non-invasive method of measuring the average pressure across the light beam. Laser interferometry presents itself as a candidate technology to enable this measurement [84-90].

Figure 3.2 Planar optical wavefronts impinging on a refractive index variation due to an ultrasonic disturbance where ‘C’ represents compression and ‘D’ dilation.
3.3 Laser Interferometry

The principle of interferometry is based on the optical interference observed when two coherent light beams, reference and object, are made to coincide onto a photodetector. From the interference detected, the frequency \( (f_o) \) and phase \( (\phi_o) \) difference between the object and reference beam \( (f_{REF} & \phi_{REF}) \) can be discerned. The ensuing frequency difference is proportional to instantaneous velocity and the phase difference proportional to the instantaneous displacement of a moving object. Phase variations of sub-degree proportions are readily attainable using interferometry; hence this technology presents itself as a useful measurement tool in quantifying phase alteration due to acoustic fields. Interferometers are invariably one of two types: homodyne or heterodyne.

3.3.1 Homodyne Interferometers

A homodyne interferometer is a single frequency device in that it does not involve any frequency shifting of either the reference beam or the object beam through the use of a Bragg cell. Figure 3.3 illustrates a basic heterodyne interferometric system that can be used for pressure detection; a homodyne system would be similar with the Bragg Cell removed. Letting the amplitude of the output at the photo-detector be

\[
A = A_o + A_{REF}
\]  

3.7

where the amplitude of the signal and reference beams can be given as
\[ A_O = \frac{A_L e^{i(\phi - \phi_O)}}{2} \]  \hspace{1cm} 3.8

\[ A_{REF} = \frac{A_L e^{i(\phi_{REF} - \phi_O)}}{2} \]  \hspace{1cm} 3.9

where \( A_L \) is the amplitude of the source. The phase shift introduced to each beam will be proportional to the optical distance travelled, which is equal to

\[ \varphi = \frac{2\pi L}{\lambda_L} \]  \hspace{1cm} 3.10

where

- \( \varphi \) = generic phase shift
- \( L \) = the optical distance
- \( \lambda_L \) = the wavelength of the laser light.

The resultant intensity at the detector, \( I \), is equal to the magnitude of the combined amplitude squared, hence

\[ I \propto |A|^2 \]  \hspace{1cm} 3.11

Combining Equation 3.11 with Equations 3.8 & 3.9 gives the intensity at the detector as

\[ I = I_L \left[ 1 + \cos(\phi_{REF} - \phi_O) \right] \]  \hspace{1cm} 3.12

As the two waves are generated from a common source they will have the same phase at the origin, assuming the internal path length is constant, the phase difference between the two beams, \( \delta\varphi \), is solely representative of the displacement of the object (see Figure 3.3), and can be described as
\[ \delta \phi = \frac{4\pi D}{\lambda_L} \] 3.13

where \( D \) = displacement of scanned object
\( \delta \phi = \phi_{REF} - \phi_O \)

According to Equation 3.13 any phase change in the object beam will be due, ideally, to the movement of the object under investigation. However, this system is very sensitive to any changes in path length (displacement between object and interferometer head) and any movement of the reference mirror from its ambient position is also detected. Hence, this reception of unwanted vibrations can lead to inaccurate displacement results.

Assuming the object is moving with respect to time with a velocity \( \nu \) then its position at any time \( t \) is given by

\[ D(t) = \nu t \] 3.14

where \( D(t) \) = displacement at a given time
\( \nu \) = velocity
\( t \) = time.

Thus, the phase difference becomes time-dependent

\[ \delta \phi = \frac{2\pi 2\nu t}{\lambda_L} = 2\pi f_D t \] 3.15

where \( f_D \) = Doppler frequency induced by a moving object.

Substituting Equation 3.15 into Equation 3.12 provides
If the motion of the object under investigation is harmonic, the interference between the object beam and reference beam will produce a frequency shift at the detector equal to the Doppler shift, $f_D$. However, this frequency is independent of sign i.e. the frequency is the same whether the surface is moving towards or away from the interferometer. Therefore, the Doppler frequency is proportional to the modulus of the object’s velocity, shown by

$$f_D = \frac{2|v|}{\lambda_L}$$

This illustrates the inherent disadvantage associated with homodyne-type interferometers; displacement magnitude is readily available, but phase information is not. This problem is solved through the use of heterodyne systems.

Figure 3.3. Schematic diagram of a typical interferometer set-up including the presence of the frequency shifting Bragg cell found in heterodyne type systems.
3.3.2 Heterodyne Interferometers

The presence of a Bragg cell in the reference arm of heterodyne interferometers introduces a frequency shift, $f_B$, into the reference beam, which allows the Doppler frequency, Equation 3.17, to be added or subtracted from the fixed frequency shift appropriately, hence

$$f_{MOD} = f_B \pm 2 \frac{v}{\lambda L}$$

where $f_{MOD} =$ resulting frequency at the detector

Substituting Equation 3.18 into Equation 3.16 provides

$$I = \frac{I_L \left[ 1 + \cos \left( 2\pi \left( f_B + \frac{2v}{\lambda L} \right) t \right) \right]}{2}$$

Any movement of the object is now no longer purely a function of the Doppler frequency; it is a directional variation of $f_{MOD}$ from its steady state value $f_B$. Heterodyne interferometers are often referred to as directional devices due to this inherent property.

Bragg cells are typically acousto-optic cells, although they can be rotation diffraction gratings or electro-optic cells, controlled by a RF generator in order to modulate the reference signals. The most efficiently produced shift frequencies are of the order of 40 MHz [95].
**Velocity Decoder**

The detection circuitry available for interferometry is such that it can provide an output proportional to either the velocity or displacement of a moving object. The velocity decoder operates in a similar manner to FM radio demodulation, converting the velocity dependent Doppler frequency into an AC voltage. However, unlike FM demodulation the demands on the demodulators used in interferometry are considerably greater due to the significantly higher frequency deviation utilised (75 kHz in radio compared to 32 MHz in interferometry). This places a limit on the bandwidth of the system of a few megahertz, as the response of the demodulator at higher frequencies is no longer linear for constant displacements. In addition, this lack of bandwidth results in problems in the acquisition of the short transient motions such as impulse responses. Nevertheless, this method of signal processing does ensure that the system is very resilient to low frequency ambient noise.

**Displacement Decoder**

It is also possible to discern displacement information from the interference signal by comparing the phase discrimination between the RF driver signal for the Bragg cell, and the modulated signal at the photo-detector. Given this, for a phase difference of $\pi$ between the two signals a displacement of 316.5 nm ($\lambda/2$) is obtained. A much higher bandwidth is available from this type of detection, but with a reduced measurement range. Figure 3.4 illustrates the different signals available from the interferometer.
As both decoder options are available for the interpretation of interference signal, each with its own merits, accurate measurements depend on the choice of decoder. It should be noted that, under certain conditions, the value for displacement can be extracted through the velocity information and vice versa, via the relation

\[ v(t) = \frac{dx(t)}{dt} \]  

where \( x(t) \) = displacement of moving object.

### 3.3.3 Physical Limitations

There are certain limitations inherent within the interferometric process due to specific requirements by the transmission and sensory apparatus. Some of these limitations can be restricted in their overall effect on the performance of the system; others cannot. One of the major limitations of the technique is the dependence on the
reflectivity of the vibrating surface and the intensity of the reflected light that returns
to the aperture.

For interferometry to operate adequately, a certain percentage of light emitted by the
aperture must be received by the sensor head in order for a measurement to be
established. Poor reflectivity, either due to the surface being uneven causing the
scattering of light or the surface not being opaque can result in the desensitising of the
measurement process or facilitate an increase in error occurrence in data received.
The simplest solution to this potential problem is to ensure that the reflecting surface
is neither transparent nor unduly rough. Any non-opaque surface can be coated with
reflective material or reflective paint in order to maintain signal integrity. It should be
noted that despite the above problems with non-reflective and coarse surfaces the
sensory equipment requires as little as 2% of the transmitted energy to be returned
[96]. The quantity of reflected light may become more of an issue when several
boundaries are interspersed along the path of the laser light; this possibility will be
examined later in Section 3.7.

3.3.4 Quantifying Phase Change using Interferometry

As has been discussed in Section 3.2.1, pressure caused by the oscillation of an ideal
piston like transducer can be described as a sinusoidal function, represented in
Equation 3.1. As a consequence, the change in a medium’s pressure due to this wave
will also be a harmonic function.
\[ \delta p(x, y, z, t) = \hat{P}(x, y, z) \sin \left[ \omega \left( t - \frac{z}{c} \right) + \theta_c \right] \]  

where \( \delta p = \) change in ambient pressure

\( \theta_c = \) Constant phase front

Accordingly, the phase change induced by this pressure variation will also be harmonic in nature as represented by

\[ \phi(t) = \frac{2 \pi \mu_{OP} \delta p(t) L}{\lambda} \]  

where \( \mu_{OP} = \) piezo-optic co-efficient.

Figure 3.5 shows the beam from the laser interferometer passing through a plane perpendicular to an acoustic field generated by a transducer. From the illustration it is evident at a distance \( z = z_1 \) and owing to the small spot size of the laser (\( \approx 40 \mu m \)), pressure becomes a 1D harmonic function in the x-direction only, thus

\[ p(x, y, z, t) = p(x, t) \]  

As the beam of light traverses the sound field of width W, the phase change induced upon emergence from the field will be representative of the average pressure along the path of the light.
Therefore Equation 3.22 will now contain an integral, with phase change becoming proportional to the integral of pressure along the light path, as such

\[
\phi(t) = \frac{4\pi \mu_{op}}{\lambda} \int p(x,t) dx \quad 3.24
\]

Due to the nature of interferometry the light beam will negotiate the pressure field twice, there and back, doubling the induced phase change. Assuming pressure is harmonic in nature, the resultant phase change induced by it will also be harmonic, hence

\[
\phi(t) = \frac{4\pi \mu_{op}}{\lambda} \int p(x) \sin \left[ \omega \left( \frac{t - Z}{c} \right) + \theta_c \right] dx \quad 3.25
\]

where \( \theta_c = \text{constant (planar) phase} \)
By letting

$$\frac{4\pi \mu_{op}}{\lambda} \int_{W} p(x)dx = \varphi_{p}(x) \quad 3.26$$

It can now be stated that

$$\varphi(t) = \varphi_{p}(x) \sin \left[ \omega \left( t - \frac{z}{c} \right) + \theta_{c} \right] \quad 3.27$$

Therefore, the phase change caused by pressure will have a sinusoidal variation with time and as a result, will cause additional modulation to the intensity of light incident upon the photo diode proportional to pressure. Accordingly, Equation 3.12 can be modified to

$$I = I_{L} \left[ 1 + \cos(\varphi(t) + \varphi_{REF} - \varphi_{O}) \right] \quad 3.28$$

As the path length of both the reference arm and the object arm of the interferometer will remain constant and there is no motion of the reflector, the only modulation of intensity will be solely due to pressure variations in the medium. Hence, letting $\varphi_{REF} - \varphi_{O} = \varphi_{C}$, intensity at the detector can be written as follows

$$I = I_{L} \left[ 1 + \cos(\varphi(t) + \varphi_{C}) \right] \quad 3.29$$

When this intensity information in Equation 3.29 is demodulated (by either the displacement decoder or the velocity decoder) the output will purely be a function of the phase variation due to pressure. Therefore, the magnitude and frequency of this variation will depend on the magnitude and frequency of the pressure wave.
3.4 Radiated Fields of Transducers

Over the past century a great deal of literature has been dedicated to the nature of sound generated by planar and non-planar radiators. Lord Rayleigh’s theory of sound [1] was a pioneering text in the development of the theory to describe the various observations that had been made at and around that period. Many of these theories remain embedded in the foundation of more recent research, the prime example of which is the Rayleigh Integral. A basic overview of some of the more simple concepts involved in describing acoustic fields is outlined in the following sections in order to provide a stable platform for the basic theory behind the field prediction techniques used in this work.

There are several assumptions that will be made when reviewing the nature of radiated fields produced by ultrasonic transducers, these are: the vibrating surface is assumed to be circular and planar with uniform motion at all points on its radiating aperture; the aperture is housed within an infinite, perfectly reflecting rigid surface known as a baffle, the source excitation will be single frequency, continuous wave and the transmission medium is isotropic, homogeneous and unbounded. A thorough overview of radiated fields of ultrasonic devices is presented in D.A Hutchins and G. Hayward’s chapter in Physical Acoustics [97], and also in Kinsler & Frey’s excellent text [98].
3.4.1 Near Field & Far Field Regions

The focus of this Section will be on axial profiles in an effort to provide a more intuitive background to a specialised and complex subject matter. Lateral beam profiles exhibit certain similarities to axial ones; however, within the context of near field/far field discussions they are not the pertinent issue. Given this restriction, the pressure field along the beam axis generated by a device under the established sanctions will exhibit a series of maxima and minima along the profile perpendicular to the front face. This peculiar attribute compels the field to be described in terms of two distinct regions, namely the near field and far field regions or Fresnel and Fraunhofer zones respectively [99].

By letting $z=0$ be the vibrating surface, moving out along the beam axis a final maximum is reached where these pressure oscillations in the Fresnel zone cease. This distance is commonly known as the near field/far field boundary, $N$, and is approximated in Equation 3.30. After this point, as $z$ increases, the pressure amplitude will fall steadily with an amplitude profile reminiscent of that generated if a point source were situated at the boundary. Both regions are illustrated in Figure 3.6.

$$N = \left(\frac{d^2 - \lambda^2}{4\lambda}\right)$$  \hspace{1cm} 3.30

where

\begin{align*}
d &= \text{diameter of aperture} \\
\lambda &= \text{wavelength of longitudinal waves in medium}
\end{align*}

The presence of these oscillations in the near field followed by a more uniform decrease in pressure can be preliminarily described by the contribution of two types of
emissions; plane waves and edge waves. The former of the two can be portrayed as the wave emanating from the disc and the latter as an inverse wave emanating in all directions from the source/baffle boundary. At axial positions where a maximum exists, the two components are in phase and hence due to constructive interference the two amplitude magnitudes will be added together. Given this is the case for in-phase components, out of phase components will, due to destructive interference on this occasion, have their amplitudes negated exactly. The number of maxima is dependent on the ratio of the aperture diameter to wavelength of sound, with an aperture of larger diameter producing more peaks than that of a smaller source, given the wavelength is unchanged. However, as the ratio continues to increase (\( \gg 1 \)) the radiated wave will no longer diffract and will become a plane wave.

This relationship between aperture diameter and transmitted wavelength, or operating frequency, can have a significant impact on the type of field structure one would witness were the device to be driven at a single frequency. There are three particular cases to consider; the radius is much smaller than the transmitted wavelength, the radius is much larger than the transmitted wavelength; they are both approximately equal to one another, as depicted in Figure 3.7. In the first instance, as the radiating aperture now resembles a point source, a hemi-spherical wave can be expected as the output. The second scenario is the opposite from the first, the size of the radiator compared to the wavelength is much larger, hence, the devices behaves as a plane wave emitter with no oscillations present in the near-field zone. The final scenario is the most interesting of the three, when the size of the radiator is comparable to \( \lambda \), and gives rises to the familiar near-field oscillations discussed previously. Generally, most transducers generate a field profile that demonstrates diffraction effects to varying
degrees dependent on the relative size of the radiating aperture. However, evaluating
the entire field profile for a particular device provides a much more intuitive method
of determining device behaviour than the theoretical methods outlined.

\[ \begin{align*}
\frac{a}{\lambda} &<< 1 \\
\frac{a}{\lambda} &>> 1 \\
\frac{a}{\lambda} &\approx 1
\end{align*} \]

Figure 3.6. Sample axial pressure distribution generated from an ideal circular piston source in a
homogenous medium.

Figure 3.7. Illustration of the effect aperture diameter compared to transmission wavelength can have
on the acoustic emissions.
3.4.2 Rayleigh Integral

Now that a basic concept of the anatomy of radiated fields under previously outlined conditions has been established, a method to quantify the value of pressure at the maxima in Figure 3.6 is required. It should be noted that for solely axial pressure, $P$, Equation 3.31 can be used providing all required transducer parameters are known, hence from [59]

$$P = P_o 2 \sin \left( \frac{\pi}{2} \left( \sqrt{\frac{D^2}{\lambda^2} + z^2} - z \right) \right)$$ 3.31

This equation produces the axial profile of the type illustrated in Figure 3.6.

A basis for the calculation of pressure at a point $r$ at a time $t$ in the field can be established through the Rayleigh integral for velocity potential stated in [78]

$$\varphi(r,t) = \frac{1}{2\pi} \int_S \frac{v_o \left( R, t - \frac{R}{c_o} \right)}{R} ds$$ 3.32

where $c_o = \text{longitudinal speed of sound in the homogenous medium}$

$R = \text{distance from surface to field point according to geometry}$

in Figure 3.8, $R = \sqrt{(x_r - x_o)^2 + (y_r - y_o)^2 + (z_r - z_o)^2}$

$\varphi(r,t) = \text{scalar velocity potential at some point}$

$v_o \left( R, t - \frac{R}{c_o} \right) = \text{instantaneous particle velocity at the source.}$
The integral in Equation 3.32 is essentially a statement of Huygens principle, which says that the final velocity potential at a point is the summation of a series of point source contributions, $ds$, where each point is assumed to contribute to the total field at any given position in that field.

The acoustic pressure, $P$, is simply found by the product of load medium density, $\rho_o$, and the partial derivative of the scalar velocity potential $\varphi$, as such

$$P = \rho_o \frac{\partial \varphi}{\partial t}$$  \hspace{1cm} 3.33

Hence, we can now state that the acoustic pressure at some point $r$ in the load medium, shown in Figure 3.8, can be calculated by a combination of Equations 3.32 & 3.33, providing

$$P(r,t) = \frac{\rho_o}{2\pi} \frac{\partial}{\partial t} \int \frac{1}{R} \frac{1}{c_o} \varphi(R, t - \frac{R}{c_o}) \, ds$$  \hspace{1cm} 3.34

Therefore, if the instantaneous velocity of a radiating aperture is known and transmission medium density in a state of equilibrium is also known, pressure at any point in the field can be evaluated.
3.4.3 Surface Displacement and Field Prediction

As has been established from the discussion of Rayleigh Integral in Section 3.4.2, if the radiating surface can be broken down into small discrete segments and the vibration for each sample measured, and assuming a hemi-spherical wave is emitted from each point on the surface, the pressure at any point in the medium can be calculated using Equation 3.34. This adaptation of Huygen’s principle and surface displacement measurement for field prediction from practical devices was utilised in Benny et al’s article [78]. The surface can be divided into elemental sources by means of an accurate scanning system comprising of two sub-micrometer micro-positioning in orthogonal directions. Combining this with the type of laser interferometry discussed in Section 3.3.2, the absolute displacement and relative phase of each element, excited at a single frequency, can be measured and stored. This information then can be used as the velocity potential function in a discretised version of the Rayleigh field integral modified to include absorption losses for a specific medium. From Benny, if the transducer operates at a single frequency into a lossy environment,
the surface velocity function will undergo the frequency dependant influence of the attenuating load [78], hence

\[ v(r, t) = a(r)e^{-dR + j\omega t} \]

where \( a(r) \) = the complex velocity strength at \( dS, r = |r(X_M, Y_M)| \).

\( \alpha \) = the frequency dependant medium attenuation co-efficient

The pressure distribution \( P(r, t) \) can then be calculated

\[ P(r, t) = \frac{\rho_0}{2\pi} j\omega e^{j\omega t} \int_{S} \frac{1}{R} (a(r)e^{-j\beta R}) ds \]

where \( \beta = \alpha + jk \)

For computational purposes Equation 3.35 is discretised and the directional response from an ultrasonic detector is incorporated. The pressure at any point in the field is then given by

\[ P_\alpha(X_p, Y_p, Z_p) = \frac{\rho_0}{2\pi} \sum_{M=1}^{N} \left( a(X_M, Y_M) \exp(j\phi(X_M, Y_M) - \beta R) dX dY \right) \]

where \( \phi(X_M, Y_M) = \) measured relative phase at elemental point on the transducer surface, related to the amplitude

\[ a(X_M, Y_M) \]

\( D \) = diameter of the ultrasonic detector.

\( \theta = \arctan(\sqrt{(X_r - X_p)^2 + (Y_r - Y_p)^2} / Z_p) \)

\( N \) = number of nodes used to discretise the surface.
Therefore, by creating an array of positional data coupled with the corresponding acoustic pressure for each point, the field can be visualised in terms of absolute pressure for a source driven with a continuous wave signal.

At present, there are two surface displacement measurement systems in operation at Strathclyde for the investigation of transducer motion and each has their own merits. The older system features a Polytec OFV-302 Helium-Neon laser interferometer in conjunction with a Polytec OFV-2700 controller incorporating an OVD-030 displacement decoder. The average spot size of the laser is under 40 µm providing the possibility of high resolution scans as the active aperture of most ultrasonic devices is >15mm. The decoder is capable of resolving out of plane motion at sub-nanometre levels under ideal conditions with a maximum range of ± 75nm. Measurement bandwidth extends from 25 kHz to 20 MHz preventing the introduction of any low frequency noise components into the information signal. Sample position is controlled by two Physik Instrumente (PI) 300 mm linear positioning stages capable of 0.1µm resolution. The procedure is automated through a vendor PC, which controls data collection and position instruction.

The alternative system is a substantial upgrade on the previous one, albeit based on the same fundamental principles outlined in Section 3.3. The Polytec PSV 300 scanning vibrometer consists of a Helium-Neon source with the object beam capable of being scanned rapidly across a surface by two mirrors attached to precision motors. This angular manipulation of the beam along both lateral axes allows for a greatly improved scanning time over the mechanical manipulation of the device used in the older system. Once again, laser spot size can be focused to below 40 µm. An
integrated software package, Vibsoft [100], controls the scan initiation, data acquisition and data presentation, which is available as an animation of device motion. However, this system utilises the velocity measurement principle and as a result the decoder has an upper bandwidth limitation of 1.5 MHz (with the lower limit allowing for DC measurements), or more accurately 1 MHz due to limitations on the data acquisition components, although upgrades are available to increase this range to 20MHz. Furthermore, the use of the velocity decoder allows for very high out-of-plane motion resolution that is fully capable of accurate measurement at the pico-metre scale.

The measured magnitude and phase surface displacement profile for a 33 kHz Tonpilz device is shown in Figure 3.9 (a) & (b) respectively. Tonpilz device are simplistic sandwich transducers with ceramic rings held between an Aluminium front mass and steel backing mass with a pre-stressed bolt through the centre. They typically exhibit piston-like behaviour at their main electrical resonance mode (explained in detail in Chapter 4 – see Section 4.3.1). Due to the relatively low operating frequency of the device the newer integrated vibrometer system was used for the measurements. Absolute displacement is available from the velocity decoder through the relationship outlined in Equation 3.20 from the software.

The device was driven at its electrical resonance frequency, 33.7 kHz, continuously into an air channel at 10 Vpp, and has an active aperture diameter of 62 mm. It is clear from the Figure that the mechanical behaviour of the device is piston-like owing to the near uniform magnitude and phase distribution across the entire surface. Discrepancies in magnitude occur only in the centre and round the outer regions of the
face where the displacement is slightly greater (darker red); the reason for this will become apparent when the device is discussed in more detail in Chapter 4. The pseudo-piston behaviour is typically a desirable characteristic for most practical transducers as it allows the engineer to reliably theorise about certain aspects about the field profile generated in a particular medium. Using this device as an example and applying classical field theory for a uniform piston housed in an infinite baffle transmitting into air, it is possible to predict the last pressure maximum, the near/far field boundary, using Equation 3.30.

A two-dimensional slice (in the $x$-$z$ plane) of the simulated acoustic beam through the centre of the transducer is pictured in Figure 3.10 with the axial profile shown in Figure 3.11. This figure clearly depicts the diffracted field with oscillations present in the near field. These are expected in the Fresnel zone when the ratio between the aperture diameter and transmitted wavelength is considered; which in this case is around 0.68, categorising the device as a source that would produce diffracted acoustic field in these regions. The complicated near field generated due to the interaction between the plane waves and anti-phase edge waves is well represented by the acoustic field prediction model described previously. Furthermore, the lack of complete symmetry about the centre axis in Figure 3.10 demonstrates that the slight variations in surface displacement magnitude across the front face (Figure 3.9) can have a direct impact on the field characteristics exhibited by the device.
Figure 3.9. Experimentally measured surface displacement profile of 33 kHz Tonpilz device in air driven at 33.7 kHz with a 10 Vpp CW signal (a) Magnitude in meters ($x10^{-7}$) (b) Phase (rads).

Figure 3.10. Simulated two-dimensional beam profile generated from surface displacement information and field prediction model for a 33 kHz Tonpilz radiating into air.

Figure 3.11. Predicted axial pressure profile generated from the surface displacement information and field prediction model, for the 33 kHz Tonpilz device radiating into an air environment.
3.5 Tomography & Reconstruction

Tomography refers to the cross-sectional imaging of an object from either transmission or reflection data by interrogating the object from many different directions. It is a technique that has and continues to be used to as a non-destructive means of examining the interior of an object in a diverse range of industries. Computerised Tomography (CT) has its foundations in the medical use of X-ray tomography [101] and, in its simplest form, is based on the idea that x-rays travel in straight lines through an object. The attenuation of an X-ray as it passes through an object provides relevant information attaining to the internal make-up of that object i.e. varying local densities within an object will infer different amounts of attenuation of X-ray energy. These differing amounts of attenuation provide contrast in an image through Beer’s law, which relates incident intensity with final intensity [102] along a path through the object. In effect, the reconstructed image will be a 2D distribution of attenuation $\mu(x,y)$. If the type of energy used was coherent, monochromatic light and the object transparent, by inference this technique could be adapted for the use of laser light with the phase of the light being the equivalent to attenuation and the 2D distribution being one of refractive index.

3.5.1 Fourier Slice Theory

Assuming the object under investigation is a non-diffracting source, i.e. parallel energy beams pass through the object without being subjected to any diffractional effects, the application of the Fourier slice theory provides the most intuitive means of estimating the object composition from the projection data. By firing a number of
parallel beams through an object represented by a 2D function \( f(x,y) \), a projection, \( P_{\theta}(t) \), can be formed at an angle \( \theta \) to the \( x \) axis shown in Figure 3.12. The projection described in Equation 3.37 forms the basis for the Fourier slice theory, which states: a one dimensional Fourier transform of a parallel projection \( P_{\theta}(t) \) of an image \( f(x,y) \) taken at angle \( \theta \) gives a slice of the 2D transform \( F(u,v) \) (shown in Equation 3.38), subtending an angle \( \theta \) with the \( u \) axis. In other words the Fourier transform of \( P_{\theta}(t) \) gives the values of \( F(u,v) \) along the line BB in Figure 3.12.

\[
\begin{bmatrix}
  t \\
  s
\end{bmatrix} = \begin{bmatrix}
  \cos \theta & \sin \theta \\
  \sin \theta \cos \theta & \cos \theta
\end{bmatrix} \begin{bmatrix}
  x \\
  y
\end{bmatrix}
\]

Figure 3.12. Illustration of the Fourier slice theorem applied to an object in the \( x\)-\( y \) plane.
The theorem goes on to say that if an infinite number of projections, \( S_\theta(\omega) \), are taken then \( F(u,v) \) is known at all points in the \( uv \)-plane. The object function \( f(x,y) \) can then be reconstructed using the 2D inverse Fourier transform, \( 2\text{DFT}^{-1} \), of \( F(u,v) \)

\[
P_\theta(t) = \int f(x,y)ds
\]

\[
F(u,v) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} f(x,y) \exp[-j2\pi(ux + vy)]dxdy
\]

Therefore, through the Fourier Slice theorem, the 1D transform of a projection, described in Equation 3.39, is the equivalent of the 2D transform of the object function, described in Equation 3.38.

\[
S_\theta(\omega) = \int_{-\infty}^{\infty} P_\theta(t) \exp[-j2\pi\omega t]dt
\]

where

\( S_\theta(\omega) = F(u,v) \)

\( u = \omega \cos \theta \)

\( v = \omega \sin \theta \)

\( \omega = \text{spatial frequency} \)

A comprehensive and intuitive derivation of the Fourier slice theorem can be found in Kak and Slaney’s excellent text [103].

The outcome of Equation 3.39 is that by taking projections of an object function at numerous angles, \( \theta_\var{\theta} \), and applying the Fourier transform to each of these, it is possible to determine the values of \( F(u,v) \) on radial lines as shown in Figure 3.13.
Hence, the reconstruction of $f(x,y)$ can be achieved through the implementation of the inverse two-dimensional Fourier transform, as such

$$f(x,y) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} F(u,v) \exp[-j2\pi(ux + vy)] dudv$$  \hspace{1cm} 3.40

However, practical implementation of the Fourier slice theory in its current form is impossible as only a finite number of projections ($k$) of a restricted number of samples ($N$) can be taken. Therefore, $F(u,v)$ is only known at a finite number of points along a limited number of radial lines. Consequently, an interpolation technique is required in order to swiftly and accurately use Equation 3.40 to provide a satisfactory reconstruction of $f(x,y)$. The most common technique of interpolating these radial points to points on a square grid is the Filtered Back Projection (FBP) algorithm. This method has been used previously by O. Bou Matar et al and others [70-74, 87-89].

![Image of projection data in the spectral domain with the intersection between radial circles and lines representing values obtained from the Fourier Transform of the angular projections.](image)

Figure 3.13. Collection of projection data in the spectral domain with the intersection between radial circles and lines representing values obtained from the Fourier Transform of the angular projections.
3.5.2 Filtered Back Projection Algorithm

The algorithm that is currently used in the majority of applications of straight-line tomography is the Filtered Back Projection algorithm. It has been shown to be both accurate and amenable to fast implementation through FFT functions. The algorithm itself can be derived from the Fourier Slice theorem, with the principal component being the conversion of the inverse Fourier transform (Equation 3.40) into polar coordinates and changes in its integration limits. For a thorough examination of the mechanics behind the formulation of the algorithm, Kak and Slaney’s text on Computerised Tomographic Imaging [103] is recommended. The fundamentals of the FBP algorithm will be examined in order to gain an understanding of the mechanisms involved in generating accurate reconstructions for application in experimental pressure measurement used in Section 3.6.2.

By referring to Figure 3.13 once again, it can be clearly seen that the density of intersections between the radial circles and projection lines lessens as the focus moves outward from the origin. This implies that there is a greater likelihood of error being incorporated in the calculation of higher frequency components by the Fourier slice theory, which would suggest the partial degradation of any subsequent reconstructed image of the object. As the name implies the FBP algorithm consists of two parts; filtering and back projection. It is this filtering process which is used to eliminate the potential errors associated with higher frequency components attained through the Fourier Slice theory.

The filtering aspect can be more easily understood when considered as the simple weighting of each projection in the frequency domain. This weighting, illustrated in
Figure 3.14(a), is used to estimate a pie shaped wedge of the objects Fourier transform, shown in Figure 3.14(b), which is what is really desired from the reconstruction process. It is best explained as the taking of the value from the Fourier transform of $S_o(\omega)$ and multiplying it by the width of the wedge (W) at spatial frequency $\omega$. The width of the wedge is inherently dependent of the number of projects ($k$) therefore, at a given $\omega$ (in the $u,v$ plane), the width of the wedge can be described as the following

$$W = \frac{2\pi|\omega|}{k}$$

Equation 3.41 modifies each projection to conform to the filter shape shown in Figure 3.14(c) and produces what is known as a filtered projection, $Q_o(\omega)$. It can be seen from the illustrations that the higher frequencies are more heavily compensated, this relates back to the fact that the majority of error contained within an unfiltered reconstruction can be accounted for by interpolation errors encountered when dealing with higher frequencies i.e. points further from the centre of the projection data, as shown in Figure 3.13.

Lastly, the back projection aspect of the algorithm is applied to the filtered projections discussed previously. The final reconstruction is found by adding together the two-dimensional inverse Fourier transform of each weighted projection, succinctly: the

Figure 3.14 (a), (b) & (c). Illustration of filter shapes (wedges) in the frequency domain used in the FBP algorithm. (a) shows the ideal scenario where all points in for each projection are known (b) shows what is obtained in practice and (c) shows the result of the weighted filter where the wedges have the same mass as those in (a).
resulting projections for different angles of $\theta$ are added to form the estimate of $f(x, y)$. This reconstruction process can be undertaken and completed very quickly as each projection involved only evaluates the values of the Fourier transform along a straight-line, depending on $\theta_i$. This step is known as back projection since it can be envisaged as the spreading of each filtered projection over the image plane. Obviously the more projections measured, the more accurate a reconstruction can be obtained. The final reconstructed picture of $f(x,y)$ can be interpreted as the summation of all filtered projections measured, and can be represented as

$$f(x, y) = \frac{\pi}{k} \sum_{i=1}^{k} Q_{\theta_i} (x \cos \theta_i + y \sin \theta_i)$$

These scanning and reconstruction techniques described facilitate the non-invasive investigation of acoustic fields in several ways; the phase delay induced by an acoustic wave in light is analogous to the absorption experienced by an x-ray passing through objects of varying density; the assumption of parallel, non-diffracting beams, required for the procedure to be accurate, holds well for the use of laser light under certain conditions, and the FBP reconstruction algorithm can be implemented to quickly produce images of refractive index variation, and hence pressure, throughout a selected acoustic plane.
3.6 Tomographic Scanning Arrangement

In order to utilise straight ray tomographic techniques and the properties of the acousto-optic effect, a customised scanning facility has been developed. Initially, this system had several basic requirements; housing for a suitable laser interferometer and reflector in addition to controllable motion in the vertical, horizontal and rotational axes. The system must also have sufficient range of motion to investigate the often extended fields associated with lower frequency (longer wavelength) applications in an air channel. Free-field conditions must also be maintained to ensure accurate comparisons with any theoretical fields generated i.e. no reflecting surfaces, draught-proof environment, and, ideally, temperature and moisture control [78].

3.6.1 Experimental System

The range of interferometry equipment available for the tomographic set-up is as follows:

- Polytec OFV-3001-33: Modular controller with voltage output available from 2 BNC ports
- Polytec OVD-30: High frequency displacement decoder
- Polytec OVD-02: 4 range wide bandwidth velocity decoder
- Polytec OVD-01: 5 range high linearity velocity decoder
- Polytec OFV-303: high sensitivity sensor head (HeNe laser source)
- Polytec OFV-310: Remote focus module

The OVD-30 displacement decoder has a bandwidth ranging from 50 kHz to 20 MHz, but has a restricted measurement range of ± 75nm [104]. The OVD-02 velocity...
decoder has a bandwidth ranging from 0 to 1.5 MHz and a measurement range limit velocity of 20 m/s. The OVD-01 velocity decoder has a limited bandwidth, 50 kHz, but a very high measurement resolution of 0.3 µm/s [105], which corresponds to readings in the pico-metre range. It should be noted that both velocity decoders are capable of measuring DC signals (0 Hz). Consequently, the choice of decoder depends on two main criteria; frequency range and desired measurement resolution. When using the 33 kHz device bandwidth becomes the principal specification as it is below the lower limit of the displacement decoder, therefore either of the velocity decoders would be suitable for this application.

The vertical (acoustic axis) and rotational planes of motion required are provided by a Phoenix ISL custom built tomographic scanning frame, incorporating two Baldor motors controlled by a EuroCard controller, connected to a PC by a RS 232 serial link, running the MINT controller code. The motors are resolvable to less than one degree for the axis of rotation and less than 0.5 mm for the vertical. A PI M521.DG precision linear positioning stage controlled by a PI C842 ISA controller provides the accurate absolute positioning required for the horizontal plane. The stage itself is capable of 1µm resolution with up to a 10 Kg load fixed onto it. Additionally, the reflector for the interferometric object beam operates on the same motor that controls the vertical positioning, hence their motion is suitably synchronised. It should be noted that the lateral axis was not part of the original frame but was incorporated at a later date in order to provide the horizontal motion essential for the tomographic approach described in Section 3.5.
The OFV-303 sensor head was attached to the linear stage to allow parallel laser samples to be taken through the object. In an attempt to minimise atmospheric interference in the scanning process the whole tomographic system was enclosed within PVC walls, with only gaps for necessary cables exposed. The demodulated signal from the OFV-3001-33 controller is available from a BNC output as a voltage proportional to velocity. This output is connected directly to an Agilent 54622 oscilloscope for display and data acquisition purposes. Relative phase measurement is achievable by using the synchronisation pulse from the function generator used to drive the device, as the initial phase reading. The entire system is controlled by a vendor PC and running specifically written automated tomographic scanning and signal capturing software. This experimental arrangement is depicted in Figure 3.15 with a picture of the main components shown in Figure 3.16. Finally, a reconstruction algorithm based on the principles of the FBP algorithm was written in the MATLAB language for data-handling and image reconstruction.
Figure 3.15. Illustration of basic tomographic scanning arrangement incorporating the laser interferometer and associated equipment. The dotted arrows denote connections between components.
3.6.2 Air Coupled Tomographic Scan

It has been established that it is indeed possible to quantify the pressure in a load medium through the influence the oscillating acoustic field has on an interferometric beam traversing the field. This knowledge can now be extended in an attempt to generate images of peak pressure at various planes in the acoustic beam through tomography.

The experimental set-up is similar to that described in Section 3.6.1 with some adjustments to facilitate the tomographic procedure. As the scanning process can prove time consuming and produce large amounts of data, a balance between scanning time and resolution must be struck. Given this, in these initial attempts the number of projections was limited to 18, or 10 degree rotational resolution, with
lateral resolution being 1mm across a diameter of 80mm, hence 81 values were generated for each projection. The device under investigation was the 33 kHz Tonpilz driven CW with a frequency of 33.7 kHz at 10 Vpp, with the x-y plane examined being 95 mm (the predicted distance for the last maximum) from the device front face. Data was captured using the acquisition capabilities of the Agilent 54622 oscilloscope controlled by a vendor PC. Again the OVD-01 velocity decoder was used due to the small amplitude of the signals involved, caused by the impedance mismatch between the radiating aperture and air coupled load, and the bandwidth restriction of the displacement decoder (>50 kHz). The tomographic reconstruction of the projection data was completed in MATLAB code using the FBP algorithm and additional low-pass spatial filtering to remove the high frequency noise components. Furthermore, the field model described in Section 3.4.3 is used here to provide a comparative slice of predicted pressure evaluated from the 33 kHz surface displacement profile, shown in Figure 3.9.

Figure 3.17 (a) & (b) show the favourable comparison between the normalised simulated field generated through the acoustic profiling tool described in Section 3.4, and the normalised reconstructed measured field respectively, with the main focus and first side lobe clearly discernable in both images. The main lobe in the measured profile can be seen to be slightly broader than that of the predicted field with a degree of dispersion in the side lobe also present in the experimental profile. The discrepancies between the two profiles may be due to transient conditions in the load medium, i.e. variations in ambient pressure and temperature, air currents and environmental noise.
The 2D images, Figure 3.18 (a) & (b), depict relative pressure intensity in an area 80×80 mm (although zero padded to 120×120 samples for a degree of contrast and more accuracy in the reconstruction algorithm) with higher intensities shown in red and lower in blue. This type of presentation makes the comparison between the two more convenient as the full profile is available at a glance. It also clearly demonstrates the differences between the theoretical and practical fields i.e. the variation in side lobe characteristics. Given this, the scattering of acoustic energy towards the edge of the experimental data i.e. around the ± 40mm points compared to that of the simulated scan, is more pronounced than in the 3D image. In the simulation the energy is almost completely contained within a range around ± 35 mm, suggesting a fairly directional beam as would be expected for this device due to $a/\lambda \approx 1$. However, with regards to the experimental projection using tomography, the distribution of energy is distinctly less compact suggesting more dispersion than was predicted. This characteristic can be attributed to the previously mentioned unstable atmospheric conditions hence emphasising any dispersion incurred during the scan.
Figure 3.17. Comparison of (a) 3D simulated pressure plane 95 mm from device front face and (b) 3D measured pressure plane using tomography 95 mm from front face. Both are for the 33 kHz Tonpilz device.
Figure 3.18. Comparison of (a) 2D predicted pressure plane 95 mm from device front face and (b) 2D measured pressure plane using tomography 95 mm from front face. Both are for the 33 kHz Tonpilz device.
Figure 3.19 (a) and (b) pictorially represent the simulated and measured projection data respectively for easy comparison of the two. It should be noted that the simulated projection data is actually a reversal of the tomographic process with the starting data set in this case being an image of relative pressure from simulation. This is known as the Radon transform of the pressure profile [103]. As can be seen from the figure, the simulated projection data is virtually symmetrical about 90 degrees (Figure 3.19 (a)), whereas the measured data (Figure 3.19 (b)) is very much asymmetrical. Despite this deviation both show a greater intensity around the lateral 0 mm axis. This similarity is expected as at the plane 95 mm from the front face, the energy in the main lobe is concentrated around the centre of the device. Starting from a point in the centre of the x-y plane and moving outward toward the edge of the beam in either direction, the magnitude of pressure will fall steadily as the emphasis is shifted from the main lobe. The difference between the two images in terms of the measured data not being symmetrical may be partly explained by the fact that the face of the transducer may not have been aligned in parallel with the laser beam, and hence, was operating at an angle which is not perpendicular to the z axis.

One other point of note between the two projections is the increased concentration of intensity around the centre axis in the simulated profile. The dispersion of the energy in the measured profile may be caused by the spatial filtering employed to remove any spurious samples. This would result in a slightly less directional beam appearing to be generated from the device.

The surface displacement profile taken through laser vibrometry illustrates that device behaviour is almost rotationally symmetrical, thus producing a near symmetrical
beam profile in a load medium, as shown in Section 3.4.3. Therefore, it is reasonable to assume that the information obtained from every angular projection for a particular plane of pressure would be equivalent. This concept is demonstrated very lucidly in Figure 3.20 (a), where each projection for a plane 95 mm along the acoustic axis is plotted on the same axes to show similarities between the information extracted at each angle: only small deviations between each project are evident. Notwithstanding, the same technique was applied to the measured profile, Figure 3.20 (b), which does not demonstrate coherence in angular projection data to the same degree, but remains close.
Figure 3.19. Comparison of (a) 2D Radon transform (simulation data) for the pressure plane 95 mm from the front face (b) 2D measured projection data using tomography 95 mm from front face. Both are for the 33 kHz Tonpilz device.
Figure 3.20. Comparison of (a) simulated Radon transform for the pressure plane 95 mm from the front face for each angle and (b) measured projection data using tomography 95 mm from front face for each angle. Both are for the 33 kHz Tonpilz device.
3.7 Influence of Cell Wall

It has been established in Section 3.5 that equidistant parallel samples of monochromatic light through an acoustic plane are required to accurately recreate images of pressure. When the active device is contained within a vessel, as is the case with many industrial ultrasonic applications, the parallel projections are broken down by the presence of the container walls. Hence, for the tomographic reconstruction process to be re-established these projections within the vessel must be maintained. If the refractional effects caused by the container walls can be predicted for each sample, it is possible to manipulate the firing angle of the laser in a way that circumnavigates these problematic effects.

3.7.1 Effects of Refraction

The nature of light dictates that when a propagating ray encounters a material of different density than that of the incident medium at an angle that differs from the normal, it will deviate from its original course depending on the incident angle at the new boundary, as shown in Figure 3.21. The amount of refraction can be evaluated using Snell’s law, as such

\[ n_i \sin \theta_i = n_t \sin \theta_t \]

where

- \( n_i \) = refractive index of incident medium
- \( n_t \) = refractive index of transmission medium
- \( \theta_i \) = angle of incident light with respect to normal
- \( \theta_t \) = angle of transmitted light with respect to normal.
Snell’s law can be applied at every boundary a beam of light encounters providing either the transmission angle or incident angle and the two media densities (refractive index) are known. If the transmission medium is denser than the incident medium, air-to-glass for example, the transmitted beam of light will bend toward the normal and vice versa.

For a cylindrical vessel with a transparent load medium and walls, a rather complex path for a beam of light traversing the internal load emerges. Refraction will occur at four distinct locations: at boundary between the environment (air) and the container (glass or perspex), between the container and the load media (typically water), at the intersection between the load and the vessel wall once more and finally between the container and the environment again. Figure 3.24 illustrates the effect refraction has on the path of laser light when attempting to create parallel projections with a water loaded cell present. In addition, due to the nature of interferometry this path will be repeated as the light is reflected back into the sensor head. This concept is of particular note when the critical angle of a boundary is taken into consideration. Furthermore, the intensity of the light received at the photodiode must be above a threshold value of 2%, as outlined in Section 3.3.3

The establishment of parallel projections within the cell media is the principal criteria when looking to solve the problem of refraction. Given this, by working backwards i.e. setting up a series of parallel rays inside the cylindrical vessel and observing the resulting paths of refracted light, one should be able to calculate the position and angle the incident beams required when entering the cell. Generic geometric solutions
will be presented for the two most common load media in a perspex cell: air and water.

Figure 3.21. Illustration of Snell’s law of refraction for light passing into a material more dense than the incident material.

3.7.1.1 Air Loaded Cell

In order to determine an idea of how the interferometer beam would be refracted when passing through an air-loaded test cell, the angles for each point of entry and exit relative to the cylindrical walls must be calculated and then plotted. To achieve this, an evaluation version of an optics software package called Raytrace [106] was used. The program did not provide precise angles for the refractional effects but it did allow for parallel projections to be created inside the cell and for the subsequent path deviations to be observed. Figure 3.22 and Figure 3.23 demonstrate the use of the program for the re-establishment of the necessary projections for an air load, with refraction at the cell wall accounted for in the latter by varying the initiation angle and position.
The important points to note from the correction in this case (Figure 3.23) are the laser starting position required and the slight overall refractional effects incurred with an air load. Regarding the starting lateral position of the laser, it is necessary for the sensor head to be positioned at a point that is beyond the cell itself. Therefore, due to the physical limitations of the tomographic frame i.e. the laser will only be capable of moving a limited distance, there will be a limit imposed upon the size of cell which can be investigated. Moreover, this limit will vary slightly depending on the cell material.

Figure 3.22. Effect of refraction when using conventional tomography to non-invasively measure pressure inside a air-loaded cylindrical vessel.

Figure 3.23. Correction of refraction effects required in order to establish parallel projections within the cell interior for an air-loaded cell.
Importantly, the minimal refractional effects result in the creation of pseudo-parallel samples being formed within a limited range around the centre of the cell for the non-corrected instance (Figure 3.22). Given this, conventional reconstruction techniques may still function for projections within these regions.

### 3.7.1.2 Water Loaded Cell

The effect on the conventional tomographic procedure due to the presence of a cylindrical vessel, described in the previous section, is far more pronounced when water is employed as the load medium. As can be seen from Figure 3.24 the light path shows no semblance to the desired projection format in the cell interior when conventional tomographic procedures are attempted. Due to the three different transmission media traversed by the laser light a complicated path is formed. In terms of refraction correction this actually reveals some advantages over the air-load scenario.

Through re-establishing the projections in the cell interior (Figure 3.25), the light exiting the cell converges to a region that is less in diameter than the cell itself, hence, the physical measurement limitations imposed by the dimensions of the frame do not have the same influence on restricting the maximum size of a cell. In fact, it is now entirely possible to investigate cells with a diameter greater than the lateral motion available to the sensor head. However, a caveat exists in that this will only be applicable under certain circumstances limited by the motion range of the horizontal stage. Nonetheless, it is evident that the potential to non-invasively interrogate scaled-up versions of test vessels exists.
3.7.2 Scan Limitations

There are some inherent limitations associated with the modified tomographic inspection of pressure fields with sealed vessels. In particular, water-loaded cells facilitate the presence of a critical angle between the glass/air boundaries when angular correction is applied, as shown in Figure 3.25, hence reducing the potential area available for scanning. Figure 3.26 depicts the reduction in scanning diameter due to this phenomenon, where the internal diameter is effectively reduced by
approximately 35%. Furthermore, it essential that the scanning diameter is reduced as the reconstruction process converts the polar grid to a rectangular grid with sides equal to the diameter of the scan. If the scanning area was not reduced in this instance the reconstruction algorithm would include regions of pressure that are not contained within the cell interior, but are contained within the perspex wall. This overlap is not desirable for accurate measurements. This reduction in scanning area is unavoidable with the technique and unfortunately it is not possible to obtain pressure information close to the cell walls. In many applications this could be a drawback as often cavitation zones can be quite common in these regions. However, for applications where only the centre of the cell is of interest, this restriction is much less of a problem. It is important to note that probe other measurement techniques may also struggle to operate at regions close to the cell walls due to sensor manipulation difficulties. Therefore, again this is not viewed as a major limitation for the non-invasive technique developed in this work.

Figure 3.26. Top view of restricted scanning area imposed due to the effect of the critical angle at the glass/air boundary for a water loaded cell.
3.7.3 Corrective Algorithm

From inspection of Figure 3.23 and Figure 3.25 that illustrate the ray-paths required to correct for refraction, it is evident that two main cases with different requirements exist: an air loaded and a water loaded test cell. For each unique case, the two principal unknowns are laser lateral position, $Y_{LASER}$, and laser firing angle, $\theta_F$. Furthermore, correction must be generic for various cell diameters, wall thickness, wall material and transmission loads, providing all are within reasonable limits. To ensure the reconstruction algorithm will function correctly, it is essential that the parallel samples through the test cell remain equidistant despite the re-positioning of the laser head. Hence, all other variables can be calculated by working back from this initial condition and applying the laws of refraction to plot the eventual ray path. This can be achieved through the application of simple geometric principles.

According to Section 3.7.2 there is a limitation placed on the maximum scan diameter attainable due to the existence of a critical angle, $\theta_C$, at the wall/air interfaces in both scenarios. It is therefore crucial this value is calculated for each cell and the subsequent scan range evaluated from it. A procedure was created in C that calculated the angle for transmission, $\theta_T$, at the wall/air boundary and compared it to the critical angle for each parallel sample starting at the edge of the cell (away from the centre). Once $\theta_T < \theta_C$ and the maximum scan diameter, $D_{SCAN}$, was established using an iterative process, a further 10% reduction in $D_{SCAN}$ was imposed to ensure that the laser light would not be overly dispersed as it breached the boundary. This determined the maximum scan diameter reliably achievable for each generic cell.
It is at this stage where the differences between an air-loaded and water loaded cell arise. If the value of the transmission angle exceeds a certain value, which is dependent on the load refractive index, then an alternative calculation is used to evaluate laser angle and position. This case is unlikely to occur in reality as almost all practical loads will have a refractive index greater than air and close to water. Therefore, although this type of geometry is written into the scanning routine, only the water loaded scenario will be described in this Section.

Finally, through combining knowledge of exit angle, $\theta_N$, and the distance between the cell and laser at normal incidence (this can be derived from the overall cell diameter), the absolute values of laser firing angle and lateral position can be accurately calculated. Figure 3.27 illustrates the laser path for the first sample with key values described in this Section highlighted in the Figure. The complete algorithm involves a larger number of angles than are shown in Figure 3.27. Also, only a half of the cell is shown for convenience and the cell wall thickness is exaggerated for clarity.

It should be noted that an additional component is required to supply the fourth degree of motion to correct for cell refraction. This is provided by a PI M038.DG rotary stage, with a resolution of 0.1 degrees, controlled by a PI C842 ISA controller. The OFV-303 interferometer head is fixed onto the rotary stage, which is in turn fixed onto the M521.DG linear positioning stage already utilised for the lateral resolution. The modified experimental set-up for sealed perspex cell is shown in Figure 3.28. Importantly, the entire 4-axis system is synchronised so that a complete 3D data-set of internal pressure can be recorded without interruption or further input from the user.
Figure 3.27. Illustration of the formulation of a generic geometric algorithm used to compensate for the effects of refraction at the cell walls
Figure 3.28. Experimental arrangement for modified tomographic scanning procedure incorporating angular correction for refraction.
3.8 Summary

This Chapter has provided a comprehensive discussion on the origins of the visualising acoustic pressure by laser light. The physical concepts behind the acousto-optic effect within the context of quantifying pressure have been presented and explained in detail. Laser interferometry has been introduced in conjunction with basic tomographic routines and reconstruction algorithms, as a practical means of utilising the properties of the acousto-optic effect for complete non-invasive 3D field measurement. For validation purposes, accurate field prediction models based on surface displacement measurements and the Rayleigh Integral have been utilised to generate the acoustic profiles from a 33 kHz Tonpilz device operating into an air channel. Next, a complete automated scanning system constructed for the synchronised measurement of acoustic pressure fields was outlined with potential limitations of the technique highlighted and discussed. This system was then used to measure a 2D slice of pressure from the 33 kHz device 95mm from the front face in an enclosed environment under free-field conditions. Importantly, the measured profile demonstrates excellent corroboration with the simulated result, hence verifying the viability of the non-invasive measurement technique as a field profiling tool.

However, when attempting to characterise pressure fields within a cylindrical cell, the tomographic procedure employed previously in the free-field scenario is no longer valid due to cell walls refracting the laser light, breaking down the parallel samples required for accurate reconstruction. The extent of these effects in both air and water loaded test cells was evaluated using a proprietary ray-tracing package. A geometric solution for re-establishing the parallel projections through a generic cell has been
developed and was subsequently outlined and presented in an intuitive manner. Finally, a second rotational axis was added to the scanning system in order to angularly compensate for refraction, providing a fully synchronised field measurement technique for characterising 3D pressure fields within sealed cylindrical vessels.

The next Chapter describes an extensive Finite Element (FE) analysis approach to creating a comprehensive model that provides simulated pressure fields within cylindrical cells generated by both a 33 kHz and 40 kHz Tonpilz transducer. This will be used for the validate the application of the experimental procedure outlined in this Section for measuring pressure fields within sealed cylindrical vessels.
CHAPTER 4

4. FINITE ELEMENT MODELLING
4.1 Introduction

Finite Element analysis has become an invaluable tool in the research and design of ultrasonic devices and systems for disciplines such as non-destructive testing, SONAR and biomedical imaging. The accuracy of this approach has been well reported and experimentally vindicated. Given this, the ability to simulate acoustic pressures generated within complex structures was considered an important tool for the validation of the experimental procedure presented in Chapter 3. FE analysis, employing the PZFlex code [48], enables the development of a comprehensive model of both the active device and its surroundings providing the functionality to model a variety of cell structures and transmission loads, in conjunction with a selection of high power transducer technologies.

The finite element analysis code PZFlex is tailored for piezoelectric, ultrasonic and thermal modelling problems, which makes it ideal for modelling and analysing ultrasonic transducer behaviour. The code has its foundations in transient analysis making it suitable for broadband applications, and is designed specifically for wave propagation analysis [107]. The ability to generate 1D, 2D and 3D structures is readily available, although choosing the most complicated, and often assumed most accurate option, may not always yield better results e.g. a 2D model may provide equivalent results as a 3D model but with considerably less computational burden.

PZFlex places a variety of element types available at the user’s disposal; linear, elastic, isotropic for fluids and solids, anisotropic for solids and the addition of piezoelectric coupling in order to allow the simulation of piezoelectric material. In
addition, non-linear effects in fluids and tissues can be represented by incorporating the B/A properties of the medium [41], which will be discussed further in Chapter 5. Furthermore, a selection of simple circuit elements such as resistors, capacitors, transformers etc is available. These can prove useful when it is taken into consideration that the electrical drive circuitry of transducers has an effect on device behaviour, and therefore should be included in the model.
4.2 Developing a FE Model

The foundation of a good modelling approach for ultrasonic transducers begins with representing the simplest available component and progressively building upon it. By adopting this approach any discrepancies occurring during the building process can be easily isolated and remedied. For example, modelling the simple transducer in Figure 2.5 would entail creating the elements for the ceramic, then the backing layer and finally the matching layer with expected behaviour being confirmed after each stage. It is at this embryonic stage of the model that certain boundary conditions must be considered, most importantly the existence of symmetry. Identifying planes of symmetry within the model and setting the appropriate boundary conditions can significantly reduce the computational load of the model. Figure 4.1 depicts the potential symmetrical planes available within a model, however it should be pointed out that typically the greater structural complexity involved in a model, the less likely symmetry will exist.

Figure 4.1. Depiction of the various symmetrical planes available in a 3D model.
4.2.1 Material Properties

The individual components involved in creating a FEM can be analogous to the links in a chain, consisting of; FEA software, hardware, material properties and the FEA user. This established, the material properties have often been heralded as the weakest link of the chain as any significant discrepancies occurring in the model can usually be traced to inaccurate material data. There are basically two types of material available in the PZFlex code; isotropic and anisotropic, each requiring their own unique description e.g. density, longitudinal and shear sound velocity for isotropic. Many of the common materials used in the modelling process e.g. aluminium, have their relevant properties listed in textbooks and tend not to vary significantly from the standard. In addition, sufficient information can usually be ascertained from the manufacturer’s data sheets when exceptional circumstances are encountered. Nevertheless, many piezoelectric ceramics have values that are specified to ± 20% of quoted values due to batch variation. Moreover, it is possible that a degree of variation within the batch itself may exist. In this event, experimental verification of relevant properties, typically using a network analyser, is recommended [57]. Due to the construction nature of the Tonpilz device used in this work, this process was not deemed necessary in order to create an accurate model.

4.2.2 Spatial Discretisation

The meshing of a finite element model defines the resolution, and to an extent the accuracy, of the solution. For wave propagation problems, the discretisation must resolve the shortest wavelength, or highest frequency, of interest to the user. This is analogous to the Nyquist sampling theorem, which states that a signal must be
sampled at least twice during each cycle of the highest frequency of the signal. The Nyquist rate only provides the least amount of samples required for the most simplistic representation of the signal. For adequate sampling, a rate of between 8 and 20 samples per cycle is required, or in finite element modelling this translates to 8 – 20 elements for the shortest wavelength in the slowest material, although 15 is recommended by the FE community.

The balance between simulation time (computational burden) and accuracy of results remains of critical importance. Although the accuracy of results will be increased by the subsequent increase in element numbers, very little actual gain is achieved once over a certain ratio. Therefore, the experience of the user is a major factor in determining this equilibrium between desired accuracy and simulation time.

4.2.3 Types of Analysis

There are several types of analysis used within FEA: static, modal, harmonic and transient. Of these four methods, transient is the approach used by PZFlex as it provides the opportunity to obtain the same information as is available from the other approaches. For example, harmonic analysis is used for a single frequency response resulting in time dependence being eliminated from the problem leaving only terms of harmonic complex variables. Adapting this from transient analysis, the Fast Fourier Transform can be used to convert the data into the frequency domain; therefore, any particular frequency of interest can be extracted for analysis. Moreover, transient analysis has inherent advantages in that real world applications are in the time domain, i.e. non-linear propagation.
4.2.4 Element Shapes and Dimensional Analysis

Elements in PZFlex can take two different forms; standard (cartesian elements), and skewed elements. Firstly, cartesian elements, which have 90 degree corners with edges possibly being of different lengths (rectangular), are computationally inexpensive requiring little memory and can be very accurate if used correctly and result in simple to construct models. Conversely, skewed element nodes can be in any relative location (non-rectangular), are computationally expensive and significantly increase model construction time and complexity, with excessively skewed elements raising the likeliness of inaccurate results. Due to the simplicity of cartesian elements and the relative complexity of skewed elements, it is recommended that, whenever possible, cartesian elements should be used in model construction. This is valid even for curved surfaces providing element size is small enough i.e. considerably smaller than wavelength of interest. This representation of curved surfaces is known as stair stepping.

There are three modelling options at the users disposal in PZFlex; 2D plane strain, which effectively assumes an infinite model in the z direction (into the page); 2D axisymmetric, which assumes 360° rotation around the x or y axis; and finally complete 3D, where the entire model geometry is specified with no approximations (with the exception of symmetry). Illustrations of the various options are shown in Figure 4.2. In order to quickly learn model intricacies, 2D models should be used whenever the opportunity presents itself. This provides rapid simulation execution, considerably faster than 3D, and encompasses the basic design principle of starting simple and adding complexity.
4.2.5 Information available from FEA

Two main types of information are commonly extracted during post-processing of an ultrasonic transducer simulation: electrical impedance/frequency characteristics and modal shape profiles illustrating deformation of the object at a particular frequency and phase. With regards to the former, electrical impedance profiles are an important measure of performance for a number of reasons. Firstly, the simulated result is easily comparable to an experimental plot from a physical device made by an impedance analyser. Secondly, and more importantly, the impedance plot reveals the location of all resonant and anti-resonant modes. This information is crucial when the transmission and reception frequencies of a device are to be considered. Importantly, this can be used to verify material property values to some extent. With modal analysis the mechanical behaviour of the transducer at a specified frequency can be ascertained, providing an important visual aid in the analysis of device response. This functionality of the code is of particular use as it allows a direct comparison of transducer behaviour with the animations available from the Polytec PSV scanning vibrometer and other vibrational measurement tools.
Extending this principle, extraction of harmonic behaviour can also be used to generate images of pressure at a particular frequency. By including a region of the transmission load into the model, the magnitude of pressure at each node can be evaluated and plotted. This will provide the user with a simulated beam profile for an arbitrary transducer operating into an arbitrary load, with the caveat that the material characteristics are accurate. Nevertheless, this method only has advantages up to a point; simulation time increases dramatically for every wavelength of field to be calculated, and the accuracy of the simulated field is generally reduced when larger ranges are included e.g. deep into the far field (> 15 wavelengths).

### 4.2.6 Field Extrapolation

In order to prevent the exponential increase in computational burden that is required to include large regions of media in the model for pressure profiles, field extrapolation techniques can be exploited to produce field plots for homogenous transmission media without featuring an excessive number of elements. These methods involve defining a data plane within several elements of the fluid/solid boundary encompassing the transducer, with at least one wavelength of transmission media between the data plane and the top of the model, depicted in Figure 4.3. The pressure time history is then stored at this data plane for the duration of the simulation, i.e. until the model rings down. Following this, either the Fourier or Kirchhoff extrapolation method can be used for far field and full field (near and far field) regions respectively [48].
Figure 4.3. FE extrapolation procedure to generate a complete field profile in a homogenous medium without the need for representing the entire region in the FE domain.
4.3 Modelling a Low Frequency device

Low frequency ultrasound of high intensity is used for various industrial applications, as discussed in Chapter 2, due to the favourable conditions created for the generation of a cavitating acoustic field. The majority of these applications require an operational frequency of less than 100 kHz. Two common ultrasonic transducer designs that are typically considered to supply the power levels necessary to induce cavitation in a fluid medium are piezoelectric ceramic stacked transducers (commonly known as sandwich transducers) or Tonpilz devices. Piezoelectric ceramic stack transducers can be effectively tailored to operate at a desired frequency by bonding together several individual piezoelectric composite layers [108]. The alternative Tonpilz is often a more desirable choice due its more favourable operation characteristics under the potentially damaging tensile forces associated with high power operation. Due to the commercial availability of Tonpilz transducers and the reasons outlined above, these devices will be used throughout this Thesis.

4.3.1 Tonpilz Transducer

The Tonpilz transducer is a type of low-frequency transducer and its low-cost, simplicity and robustness are well known. The typical design for a piston type Tonpilz transducer, named from the German words ‘sound’ and ‘mushroom’ owing to its peculiar shape, is shown in Figure 4.4. The piezoelectric stack is assembled between a front piston mass (M₁), typically aluminium, which radiates into a transmission medium and a tail mass (M₂), which is typically steel or a similarly dense material. These transducers are also known as ‘mass spring type’ transducers, as they are analogous to the movement of a mass on a spring. Given this; when force is applied to
the mass (depending on the size of the mass and stiffness of the spring) the mass will oscillate up and down on the spring before eventually settling.

When an alternating electric field is applied to the piezoelectric stack (force on a mass), it causes the two end masses to be displaced in opposite directions (oscillation) and if the masses are equal then the displacements at each end will have the same amplitude. If, however, one mass is much heavier than the other, most of the displacement will occur at the lower mass. Additionally, an impedance mismatch exists at both faces of the device (the head material is chosen to have an acoustic impedance that is between the ceramic and water) thus facilitating a more efficient transfer of energy into the load medium. The tail mass material is chosen to have high acoustic impedance (close to that of the ceramic) as it usually radiates into an air load, therefore creating a large impedance mismatch that helps reflect energy back into the head section, thus improving device performance at the expense of reception sensitivity.

The resonant frequency, $\omega_S$, for such a configuration is given by

$$\omega_S^2 = K \frac{(M_1 + M_2)}{M_1 M_2}$$

where $K$ is the ‘stiffness’ of the piezoelectric stack (analogous to a massless spring) and is derived from the material properties of the piezoelectric stack such as; Young’s modulus, active area and active length [2].
To complete the design, the whole structure is placed under compressive stress by the placing of a steel bolt through the centre of the device. Ceramics are known to be weak under tensile forces and can rupture during the dilation phase of an activation cycle if the voltage is great enough. The pre-stress afforded by this bolt allows the ceramic stack to operate safely under high voltage excitations by keeping the device under compression, as well as maintaining intimate mechanical contact between all the components.

Historically, these devices demonstrate a limited bandwidth when operating as a longitudinal vibrator which is viewed as a constraining factor in their practicality for broadband applications. Alteration of the head mass dimensions, reducing thickness or increasing front face diameter can aid in the widening of the frequency response to a degree, but also introduces a flexural resonance mode of the head. It should be noted that impedance matching, either using an electrical network at the input of the transducer or through fixing matching layers onto the front face of the device, can be used to increase bandwidth [109]. As many of the design processes used for the
Tonpilz were based on one-dimensional (1-D) approximations, the unwanted flexural modes, often referred to as head flapping, were not well understood and dismissed as a negative influence on acoustic performance, i.e. compromising bandwidth and output power. Hence, the design procedure often employed was to ensure the flapping resonance mode was well removed in frequency from the fundamental piston mode of the transducer. With the recent advent of cheap computational resources it is now feasible to employ more complex methods to model these devices. FEA techniques allow for the full 3-D characterisation of the operational mode of the device, and as a result, utilisation of FE techniques is now becoming the standard procedure for the design of Tonpilz transducers.

The device typically has three distinct modes of operation; the fundamental piston mode, the flexural mode of the head mass, and the often-overlooked mode related to the pre-stress bolt. Several articles have been published describing techniques that can be adopted to utilise the modes other than the fundamental for increasing bandwidth performance and transmission characteristics. Yao et al [110] discuss the manipulation of the head mass and taper shape to ensure a favourable transmit voltage response (TVR) over a range of frequencies outside the two main modes. An alternative approach undertaken by Hawkins et al [111] exploits the flapping mode by lip mounting the front face, albeit for a square head shape, of the device to produce broadband, high electromechanical coupling characteristics. Both authors adopt FE modelling through the ANSYS code as their primary design approach. However, although theoretical and experimental electrical impedance profiles and TVRs are used for comparison, device behaviour at each mode could be more thoroughly verified through laser vibrometry scans of the front face. As a result, this technique
will be used extensively in this Chapter to both evaluate device performance and verify the FE models.

### 4.3.2 Analysis

Two commercially available 33 & 40 kHz Tonpilz devices from Morgan Matroc \[112\] were used throughout the investigation into internal cell pressures. Table 4.1 notes all the relevant dimensions of both devices. To ensure accurate theoretical field characteristics from any FE models used, both devices must be well represented in the FE domain i.e. demonstrate excellent correlation with experimental results in several key characterisation methods; electrical impedance profile over the main frequency range, harmonic behaviour at key frequencies and radiated field characteristics. This thorough modelling and characterisation programme is a necessary pre-cursor to the investigation of pressure fields generated by these devices in sealed vessels, with the objective of establishing the models’ accuracy.

Firstly, the 33 kHz transducer was modelled using cartesian elements with sufficient meshing to ensure the ‘stair-stepping’ of elements at the taper of the head mass was not a limiting factor in model accuracy. Also, the finer meshing allowed for detailed resolution of any energy contained in the higher frequencies. As intimate mechanical contact between the main layers (backing, ceramic and head) is maintained by the pre-stress bolt, there is no need for inclusion of the bonding layers. The copper electrodes were however included in the model. As the device is axisymmetric around the centre, the 2-D axisymmetric mode in PZFlex with symmetry around the y-axis was employed. This facilitates the cylindrical nature of the device featuring a greatly
reduced number of elements over a 3D model. Notwithstanding, a full 3D model of the device was also developed using quarter symmetry for completeness.

As it is known that these devices can be very narrowband, i.e. a great deal of ‘ringing’ occurs in response to pulse excitation, when an impulse signal is applied to the terminals a large amount of energy remains within the device over a relatively long period due to low loss in the ceramic and the impedance mismatch at the end faces. In terms of FEA, this long settling period transpires as a long simulation time. To remedy this, a degree of damping can be applied to the device through an external resistor (with a value proportional to the real part of transducer reactance) attached to the drive electronics in the FE model. This reduces the simulation time significantly without compromising the final results [48].

Table 4.1. Physical Dimensions used in the FEA of the commercially available Morgan-Matroc 33 kHz Tonpilz device.

<table>
<thead>
<tr>
<th>Aspect</th>
<th>33k Hz</th>
<th>40k Hz</th>
</tr>
</thead>
<tbody>
<tr>
<td>Front Face Diameter</td>
<td>63</td>
<td>50</td>
</tr>
<tr>
<td>Steel &amp; Ceramic diameter</td>
<td>45.6</td>
<td>38.1</td>
</tr>
<tr>
<td>Thickness of Steel</td>
<td>15</td>
<td>10.2</td>
</tr>
<tr>
<td>Thickness of Ceramic (each)</td>
<td>5.4</td>
<td>5.4</td>
</tr>
<tr>
<td>Thickness of Aluminium</td>
<td>28</td>
<td>21.8</td>
</tr>
</tbody>
</table>

The FE device is excited by a single cycle sine wave at a suitably high frequency (100kHz) to ensure energy content over the range of interest, in this case 0 – 200 kHz is available. The voltage and charge time profiles are monitored and recorded for use
in extracting an electrical impedance profile against frequency. For comparison experimental impedance profiles were obtained using a HP 4194A network analyser.

Figure 4.5 demonstrates the excellent agreement between theoretical and measured impedance in air for the 33 kHz Tonpilz around both the fundamental longitudinal resonance at 33.7 kHz, and the flexural ‘flapping’ mode at 42.5 kHz. In particular, the two main electrical resonance (ER) frequencies of the FE model correspond to the measured values of 33.7 and 42.5 kHz precisely. The third, less prominent mode exists in the measured impedance plot but is not well represented in the FE profile. This mode is typically due to the resonance behaviour of the pre-stressed bolt in place through the centre of the device. Curiously, due to the ‘ideal world’ scenario present in the FE environment, it would be expected that the simulated resonant peaks would be exaggerated when compared to the measured instances; this proves not to be the case here. Again this may attributed to the influence of the pre-stress bolt that is not represented in the model.
Figure 4.5. Comparison of theoretical and experimental electrical impedance profiles for the 33 kHz device. (---) represents the measured response (a) magnitude (b) phase.
Having established that the theoretical electrical profile of the device is in excellent agreement with the measured data, modal analysis at key frequencies, namely the longitudinal and flexural modes, can be utilised to evaluate mechanical deformation characteristics. This mechanical behaviour provides a simple yet conclusive comparison to laser vibrometry measurements of the vibrating aperture. Figure 4.6(a) and (b) illustrate the deformation of the transducer at the longitudinal extension mode, 33.7kHz, at 0 and 180 degrees phase respectively. From the figure it is clear the simulated device appears to have uniform phase and it operates in the expected quasi piston-like manner. Figure 4.6 (c) and (d) represent the mechanical deformation at the flexural mode, 42.5kHz, and it is obvious from these figures why this mode is often referred to as the bending mode: the outer regions of the surface appear to be completely anti-phase to the inner region near the centre bolt. Consequently, when driving the device in this mode, the effect on the radiated field pattern by the disrupted surface displacement motion must be taken into consideration. Importantly, animations generated from laser vibrometry scans at these frequencies confirm the theoretical mechanical behaviour at these two frequencies. This type of detailed analysis and theoretical motion characterisation would not be achievable without the use of FE software.

Figure 4.7 illustrates the absolute displacement comparison between simulated and measured displacement profiles with a driving voltage of 10Vpp in air. Excellent corroboration is achieved between the two with the increased displacement evident at the outer edges and above the centre bolt clearly discernable. The discrepancy between measured and simulated values can be accounted for by the crude fabrication procedures employed to construct these devices. Note, however, the slight asymmetric
behaviour displayed by the real device which cannot be represented in the FE model. The reliable simulation of surface displacement data can be an invaluable transducer assessment tool when it is considered in conjunction with the field modelling software described in Chapter 3.

Figure 4.6 Extraction of harmonic shapes from the 33 kHz FE model illustrating (a) fundamental - 0 degrees (b) fundamental - 180 degrees (c) flexural – 0 degrees (d) flexural – 180 degrees
Figure 4.7 Measured and simulated profile of front face displacement for the 33 kHz Tonpilz.

Figure 4.8 Measured and simulated profile of front face phase for the 33 kHz Tonpilz.
Now that the simulated electrical and mechanical behaviour of the transducer has been verified through extensive experimental characterisation, it is important to ascertain if this behaviour transfers into accurate predictions of radiated field patterns. This will allow reliable comparisons to be drawn between theoretical predictions and experimental measurements when these devices are incorporated into complete ultrasonic systems. Through Kirchoff extrapolation methods and harmonic field extraction techniques, both outlined in Section 4.2.6, theoretical representations of the acoustic field generated by the modelled devices can be generated. As transducer behaviour has been confirmed, these profiles should accurately replicate the beam profile of the practical device under free-field conditions.

Figure 4.9 shows the Kirchoff frequency domain extrapolation for the 33 kHz Tonpilz operating into an air load. The profile itself is generated in post-processing as only a small region of the transmission load is actually modelled. This technique drastically reduces simulation and increases accuracy for fields extending to a distance greater than 15 acoustic wavelengths in the load. Importantly, the profile is representative of the field generated by an ideal piston, with diffraction obvious in the near field and the last maximum occurring at around 95mm from the front face. This also compares favourably with the profile generated from measured surface displacement presented in Figure 3.10. Moreover, this technique allows the engineer to quickly simulate the field from an arbitrarily shaped aperture in a homogenous medium under a variety of excitation conditions. Nonetheless, this extrapolation technique cannot be used for simulating pressure within an enclosed space due to the presence of reflections, and the alternative harmonic extraction method must be used.
Figure 4.10 illustrates the simulated pressure extracted from a model where the entire load channel is represented in FE space. Acoustic pressure is then calculated at each node for the entire simulation time and extracted in one of two ways, depending on the excitation method. For broadband excitation (single cycle sine wave) the harmonic pressure is extracted via a Discrete Fourier Transform (DFT); for single frequency excitation (continuous wave) the model runs until the steady-state response is reached, then maximum pressure at every point is extracted. There is no significant difference in field shape between the two methods. Consequently, the agreement between the two methods presented in Figure 4.9 and Figure 4.10, and the beam profile from measured surface displacement, demonstrates the high level of accuracy achieved in the model of the 33 kHz Tonpilz transducer.
Figure 4.9. Air-coupled beam plot using Kirchhoff extrapolation generated from the FE representation of the 33 kHz Tonpilz device at 33.7kHz (fundamental).

Figure 4.10. Air-coupled beam plot using harmonic pressure extraction generated from the FE representation of the 33 kHz Tonpilz device at 33.7kHz (fundamental).
4.3.2.1 40 kHz Tonpilz

The analysis completed for the 33 kHz transducer was mirrored for the 40 kHz device. This transducer has similar characteristics to the 33 kHz device, shown in Figure 4.4, although with different dimensions as listed in Table 4.1. However, the bolt hole in the 40 kHz device runs through the entire structure i.e. the front face has a gap in the centre equal in diameter to the bolt thread.

Once more the intention is to use this device in conjunction with an ultrasonic test cell in order to quantify the pressure fields within these vessels; hence it must be modelled to the same level of accuracy as the 33 kHz model. Firstly, a comparison of the electrical impedance profile was established, shown in Figure 4.11, with the model performing well at the main resonances of interest and displaying the basic modes anticipated for such a device.
Figure 4.11. Comparison of theoretical and experimental electrical impedance profiles for the 40 kHz device.
The mechanical deformation at the first two impedance minima, 40 and 50.1kHz, are demonstrated in Figure 4.12 (a)-(d). Again, piston-like motion is inferred from Figure 4.12 (a) and (b) with the flexural mode evident in Figure 4.12 (c) and (d). Secondly, mechanical displacement at the longitudinal mode was extracted for behavioural validation against laser vibrometry scans. The fine agreement between the theoretical and experimental profile is illustrated clearly in Figure 4.13. Finally, the radiated field characteristics of the FE model were simulated using field extrapolation and harmonic pressure extraction, as in Section 4.3.2, and are shown in Figure 4.15 and Figure 4.16 respectively, with excellent agreement demonstrated. Therefore, there exist two fully characterised devices, both experimentally and theoretically, for use in the measurement and simulation of pressure fields within sealed vessels with a high level of confidence established in the theoretical model. This verification of the radiating field characteristics of FE device allows the use of the model with confidence to predict the field pressures in arbitrary structures i.e. fluid-loaded cylindrical test cells. Due to the presence of reflecting boundaries in the model, beam plots via harmonic extraction will be used to generate simulated fields within test cells.
Figure 4.12 Extraction of harmonic shapes from the 40 kHz FE model illustrating (a) fundamental - 0 degrees (b) fundamental - 180 degrees (c) flexural – 0 degrees (d) flexural – 180 degrees
Figure 4.13. Measured and simulated profile of front face displacement for the 40 kHz Tonpilz.

Figure 4.14. Measured and simulated profile of front face phase for the 40 kHz Tonpilz.
Figure 4.15. Air-coupled beam plot using Kirchoff pressure extraction generated from the FE representation of the 40 kHz Tonpilz device at 40.1kHz (fundamental)

Figure 4.16. Air-coupled beam plot using harmonic pressure extraction generated from the FE representation of the 40 kHz Tonpilz device at 40.1kHz (fundamental)
4.3.2.2 Electrode Thickness

A key physical feature of the Tonpilz transducers is the presence of a thin copper plate as the electrode between the two ceramic discs which play a crucial role in the operation of the Tonpilz transducers. The presence of the copper shim may, in fact, act as a damping mechanism on the mechanical deformation experienced by the ceramic discs as they displacement due to the drive signal. It is important to note that due to the high drive signals experienced by these devices in practice, a metallic electrode layer is a necessity to ensure electrical continuity and integrity (a silver fired electrode may degrade over time at high drive levels). A brief theoretical investigation, featuring the 40 kHz Tonpilz device, into the effect of electrode thickness was completed to assess the overall effect electrode thickness may have on the displacement profile and impedance response.

Figure 4.17 illustrates the simulated impedance response for 4 different electrode thicknesses, from 0.1 to 0.7mm in 0.2mm steps. These thicknesses were chosen as they are freely available commercially. It is clear from the figure very little effect on impedance is induced by these thickness levels. However, when the front face displacement at longitudinal mode is extracted, as shown in Figure 4.18, a substantial reduction in output displacement can be equated with an increasing electrode thickness. Therefore, it can be stated that minimising electrode thickness should favourably affect the displacement output of the Tonpilz transducer, and this parameter should be taken into consideration in any future design work. This virtual design exercise is one such example where FE techniques can be used effectively to optimise device performance through variation of physical parameters. Other
examples of such optimisation can be found in Reynolds’ article [113] which alters tail and head mass in an attempt to produce a more efficient transducer.

Figure 4.17. Impedance magnitude response for varying copper electrode thicknesses incorporated into the 40 kHz Tonpilz model.

Figure 4.18. Front face displacement for varying copper electrode thicknesses incorporated into the 40 kHz Tonpilz model.
4.4 FE Representation of a Cylindrical Test Cell

FE presents itself as the ideal solution to the difficulties involved in simulating the ultrasonic pressures generated within complex structures. Traditional field simulation techniques cannot account for the extended path taken by the acoustic energy in such complex geometries, i.e. reflections from the cell walls, holder lid and device front face. For an accurate pressure field to be simulated, all the idiosyncrasies associated with the cell that are greater than 1mm (less than 2% of the acoustic wavelength at optimum transmission frequencies) must be included in the model. Bond-lines and sealant materials are not included for the sake of brevity and diminished returns in terms of complexity versus accuracy, with the exception of the copper electrodes which have been shown to have impact on simulation behaviour.

For this work, a series of simple cylindrical test vessels are constructed from various lengths of Perspex tube and sealed with a PVC fixture. The PVC itself is machined to fit into an aluminium holder attached to the tomographic scanning frame, which is not included in the FE models. The Tonpilz transducers, described in Section 4.3, are sealed into the cell through integration with a custom made Perspex lid which is fixed onto the open end of the cell (opposite side to the PVC). This provides an acoustic resonator suitable for the creation of standing waves. Material properties for Perspex and PVC were taken from Ondacorp’s online resource [114] and used in the model. Dimensional measurements were taken from manufactured prototypes and subsequently input into the FE code. The cell FE template used in conjunction with the 33 kHz Tonpilz device and a water load is shown in Figure 4.19, with relevant dimensions listed in Table 4.2.
Figure 4.19. FE representation of a perspex cell, sealed with a machined PVC lid and featuring integration with the 33 kHz Tonpilz transducer.

Table 4.2. Listing of key dimensions used in the FE cell representation employed with the both 33 kHz and 40 kHz systems

<table>
<thead>
<tr>
<th>Aspect</th>
<th>33 kHz Size (mm)</th>
<th>40 kHz Size (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cell outer diameter</td>
<td>100</td>
<td>100</td>
</tr>
<tr>
<td>Cell wall thickness</td>
<td>5</td>
<td>5</td>
</tr>
<tr>
<td>PVC fixture thickness</td>
<td>31</td>
<td>31</td>
</tr>
<tr>
<td>PVC fixture diameter</td>
<td>77</td>
<td>70</td>
</tr>
<tr>
<td>Perspex holder thickness</td>
<td>10.5</td>
<td>18.4</td>
</tr>
<tr>
<td>Overall length</td>
<td>169</td>
<td>129</td>
</tr>
</tbody>
</table>
A similar model featuring the 40 kHz Tonpilz transducer was also created. Model parameters were defined in a manner that is conducive to variability, for example, cell width, height, material properties and load material could all be altered with relative ease without recourse to extensive modifications of the transducer models. This was implemented in order to minimise any complications involved with altering the model for any future work.

Next, FE electrical impedance profiles of both systems were compared with experimentally measured data in order to verify the operation of the systems. The comparison of magnitude and phase shown in Figure 4.20 & Figure 4.21 respectively, illustrates a main electrical resonance mode at 34 kHz with a large number of minor modes now evident. This is exaggerated somewhat in the FE profiles due to the lack of external damping factors. Figure 4.22 & Figure 4.23 show this type of behaviour is replicated in the 40 kHz system but where the main electrical resonance mode remains at 40 kHz. It should be noted that the experimental profile is damped slightly more in the 40 kHz system in comparison to the 33 kHz system.

Therefore, it is clear that these devices are particularly sensitive to loading conditions and these can have notable effects on the electrical behaviour observed. Such characterisation can be a useful resource when seeking the optimum driving signal for efficient operation in a particular system. Moreover, the correlation demonstrated also gives confidence in the ability of the FE models to predict the operational behaviour of each system.
Figure 4.20. Electrical impedance magnitude comparison between measured and FE simulation for the 33 kHz system

Figure 4.21. Electrical impedance phase comparison between measured and FE simulation for the 33 kHz system
Figure 4.22. Electrical impedance magnitude comparison between measured and FE simulation for the 40 kHz system

Figure 4.23. Electrical impedance phase comparison between measured and FE simulation for the 40 kHz system
4.5 Probe Influence

The accurate mapping of the internal fields within test vessels is often done through the insertion of a measurement probe directly into the transmission load. This purpose of this Section is to serve as an indicator to the potential affects that two common types of probe, low-frequency piezoelectric and PVDF membrane, would have on the harmonic field within a cylindrical cavity. The simulated probes are shown in Figure 4.24 & Figure 4.25 respectively, with the reader referred to Chapter 2 for more detail on both. The FE representation of the 33 kHz Tonpilz transducer and the cylindrical cell, described in Section 4.3 & 4.4 respectively, will be used throughout for brevity. A continuous wave pressure load at a single frequency of 33.7 kHz is used as the excitation signal. The simulation is then run until steady-state pressure behaviour is reached at key points in the load before maximum pressure is extracted for the field. A total of 8 probe positions are chosen for each hydrophone type increasing in 5mm steps from 20mm in front of the device, to 55mm away. Importantly, all pressure profiles are normalised to an undisturbed field, shown in Figure 4.26.

4.5.1 Low-frequency Hydrophone Probe

The probe emulated in these simulations is a Bruel & Kjaer (B&K) 8103 hydrophone with a 10mm active diameter inserted directly into the centre of the cell, shown in Figure 4.24. Due to the axisymmetric nature of the model this is the only position where a realistic representation of the probe can be modelled. The increased density of the metal holder employed to maintain the position of the hydrophone would result in an increased pressure along its length, and hence, it is necessary to zero the
pressure in this area to ensure sufficient dynamic resolution is available to discern the pressure field in the surrounding load. Due to the rounded tip of polyurethane, it was not possible to zero the pressure and hence a slight increase is noted at this boundary. However, it does not adversely affect the interpretation of the results. Figure 4.27 shows the succession of pressure profiles taken at each distance from the active face.

It is clear from Figure 4.27 that probe presence appears to have an effect on the undisturbed field profile, with distinct differences evident from merely re-positioning the probe 5mm in either direction. The high pressure nodes can be reduced in amplitude by the presence of the probe, as in the 55mm case, or enhanced in certain regions, as is the case in the 45mm simulation.

![Simulated Bruel and Kjaer 8103 hydrophone](image)

Figure 4.24. Simulated Bruel and Kjaer 8103 hydrophone in positioned in the centre of the cell model. Only probe is shown for clarity.
4.5.2 PVDF Membrane Hydrophone

Due to the favourable match in acoustic impedance between the PVDF and the water load, this type of hydrophone should prove to be less intrusive than the 10mm diameter ceramic probe. Indeed, as the PVDF layer thickness is approximately 3 orders of magnitude less than the acoustic wavelength at 33.7kHz, it is reasonable to assume that the membrane will have little impact on the subsequent pressure field. Ideally, the membrane layer would consist of a 28µm layer of electroded PVDF sandwiched between two unelectroded layers for marinisation purposes, giving a total thickness of approximately 84µm. For simulation purposes, this was approximated to 100µm and represented by 4 elements in the model. The ring-shaped aluminium holder for keeping the membrane under tension is also represented in the simulations as shown in Figure 4.25. This should still provide a reasonable insight into the overall influence a practical membrane hydrophone may have on the field profile.

Figure 4.25. Simulated PVDF Membrane hydrophone in positioned in the centre of the cell model. Only membrane probe is shown for clarity.
Figure 4.28 illustrates pressure profiles in the water load for the membrane hydrophone at various distances from the front face. From initial inspection, it is clear that the overall effect on the field is reduced with the membrane compared to the low-frequency probe. As with the B&K hydrophone, it can be suggested that the presence of the membrane actually enhances the intensity of particular regions of the field as higher levels of pressure are demonstrated. Nevertheless, although the use of a membrane hydrophone may limit the distortion of the field profile, practical constraints would still limit the effectiveness of such a device in reality. For example, manipulating the membrane, or more importantly the active area, around the three-dimensions required to generate a complete map of pressure would prove impossible without a selection of probes with varying active area positions. With this in mind, a PVDF membrane with active elements forming a 2D array pattern [115] may provide the required diversity to facilitate complete field measurement through simply moving the hydrophone along the acoustic axis. Extending this possibility further, a 3D array could be created with multiple membrane device stacked on one another. This may prove practical if the membrane layer remains thin enough, and has a mechanical impedance close to that of the transmission load, the influence on the field profile would be minimal. Moreover, due to the low operational frequencies involved in most processing applications, each PVDF layer need not have a particularly dense element count. The relatively long acoustic wavelength, 37.5mm at 40 kHz, allows a large element spacing of at most 18.725mm (half-wavelength) to obtain a suitable level of resolution. In reality an element spacing of 2mm with a similar element size should prove most adequate. Nonetheless, one drawback on such a method is the susceptibility of PVDF hydrophone to damage in cavitating fields would apply to each membrane layer in the 3D array. Interestingly, during the course of this
investigation it was found that holder shape and material had more of an influence on the field profile than the actual membrane itself. This becomes more pertinent if a 2D or 3D PVDF membrane probe is considered for field measurement in that the constituent structural materials and layout will warrant careful consideration.

This theoretical exercise has demonstrated the potential effects of probe insertion within an enclosed pressure field. The effects discussed have already been inferred in previous Chapters that outline the requirement for a non-invasive field measurement technique, however utilising the FE models in this manner reinforces this objective.

Figure 4.26. FE prediction of field profile in a cylindrical vessel for the 33 kHz Tonpilz device excited via a pressure loading function at the front face. The transducer is situated at the bottom of the image.
Figure 4.27. FE pressure field profiles for various positions of the hydrophone probe when inserted into the centre of the cell.
Figure 4.28. FE pressure field profiles for various positions of the membrane probe when inserted into the cell.
4.6 Summary

This Chapter has presented the fundamental features of Finite Element analysis with regards to modelling ultrasonic devices and systems. Key concepts such as material properties, discretisation, boundary conditions and the extraction of relevant data, have all been outlined in the attempt to define a reasonably efficient modelling approach for a variety of problems. In particular, this approach has been applied to modelling low-frequency Tonpilz transducers that are often found in high-power processing and SONAR applications. The basic structure and behaviour of the Tonpilz has been described, with reference to its electrical and mechanical properties serving as a guide to the modelling process. Both axisymmetric 2D and 3D models have been created with the 3D model showing no increase in accuracy over the 2D representation.

Extensive experimental characterisation has been completed for both 33 and 40 kHz Tonpilz devices. Laser vibrometry scans have provided a unique insight into the behaviour of these transducers at several key frequencies. This information serves as a valuable verification of FE data and provides confidence in the virtual models to represent practical operation. This concept is advanced further when the radiated fields from such devices are considered. Surface displacement data from the vibrometer allows an accurate acoustic profile to be generated in a homogenous medium, which is a powerful characterisation tool in its own right. Several methods of simulating the field profile from the virtual devices, each with their own merits, were explored and presented. From this, harmonic field extraction through modelling the entire load area of interest offers the best potential for simulating fields within enclosed regions, i.e. within a test vessel. Therefore, it can be stated that accurate
prediction of surface displacement is a direct indicator of radiated field profile accuracy.

This is further expounded through the creation of a FE model of a generic cylindrical test cell that can incorporate the Tonpilz transducers featured throughout this Chapter. Indeed, the model has been formed in such a way that the materials and dimensions of the cell are easily altered by the user. The initial design is based on accurate physical measurements of prototype cells by micrometer. Experimental impedance profiles are used for preliminary comparisons with simulations results, with reasonable agreement shown for such complex models.

Finally, a theoretical investigation was presented into the influence a low frequency hydrophone and PVDF membrane hydrophone have on the field profile. From this, it can be stated that a PVDF membrane would have the least effect on the field profile while being the simplest to implement within a vessel and, as such, any experimental measurements should be carried out with this type of sensor. However, the caveat of sub-cavitational conditions remains. Finally, a 2D PVDF array hydrophone is suggested as a measurement alternative for rapid field characterisation within enclosed vessels, although device complexity may prove to be a limiting factor.

The following Chapter builds upon the detailed FE modelling described in this Chapter and extends the field analysis to incorporate non-linear effects and the influence they have in low frequency, high-power applications.
CHAPTER 5

5. INVESTIGATION OF NON-LINEARITY
5.1 Introduction

As cavitation is frequently a desirable effect within many of these high-power low-frequency systems, with relatively large drive input voltages typically employed, these systems are commonly expected to feature highly nonlinear acoustic propagation. Consequently, this generation of harmonics would significantly alter the field profile compared to that of a linear system. However, when the short propagation distances involved are considered it is not unreasonable to postulate that these systems may remain largely linear until the onset of cavitation, in terms of classical acoustic propagation.

The purpose of this Chapter is to investigate the possible non-linear effects within such systems and evaluate their influence on the field profile. A theoretical description of non-linear propagation will be presented, with the merits of common analytical models discussed. Following this, a numerical model of non-linearity will be outlined with specific reference to the FEA code PZFlex and the advantages it presents for representing nonlinear effects in bounded fields. Next, driving equipment and transducers will be evaluated for linearity in order to disengage any effects from those formed in the transmission load. Finally, the linearity of the system will be measured via acoustic hydrophone and compared with FE analysis in order to confirm the hypothesis stated earlier in this Section.

It is often assumed that the propagation of ultrasound is a linear process i.e. that the travelling wave has constant velocity, $c_0$, and so retains its shape as it travels through the load medium. However, when higher amplitudes of input signal are employed, it
is often not difficult to generate sufficient pressures in a liquid medium to complicate linear models. When these higher pressures are achieved, nonlinear effects will become apparent and the profile of the wave will be subject to certain changes. The compressional phase of the wave will cause an increase in the local density of the medium through which it travels, and given that the velocity of sound is dependent on the momentary value of density, points in the wave of greater amplitude will travel faster than those of lesser amplitude. Thus, the positive cycle of an originally sinusoidal plane wave becomes increasingly ‘steeper’ over the course of its propagation, as illustrated graphically in Figure 5.1. Due to this distortion of the wave front in the time domain, the frequency spectrum of the wave will, as a consequence, also be altered. Spectral components, or harmonics, at $n$ multiples of the original frequency will become apparent in the Fourier transform of the temporal information. Finally, this steepening process leads to the formation of pressure discontinuities, known as shock fronts, which manifest themselves as vertical tangents in the waveform, i.e. the waveform transforms into a sawtooth profile, shown in Figure 5.1 (d).

This phenomenon has generated a great deal of interest over the past 50 years or so, spawning a variety of practical applications in the process e.g. medical ultrasound scanners using tissue harmonic imaging [116]. With the emergence of these applications comes the need for support in the form of field parameter prediction facilities, particularly when dealing with therapeutic ultrasound. Given that linear models are unable to accurately forecast the desired parameters for accurate simulation of therapeutic ultrasound, the need for an accurate simulation tool which accounts for nonlinear effects is unavoidable. Some of these tools will also have
application in the realm of high power ultrasonics and their suitability will be assessed throughout the next Section.

Figure 5.1. (a) Original linear profile. (b) 'Steepening' profile as pressure increase (c) Further distortion of waveform as higher amplitudes are achieved. (d) Final sawtooth profile.
5.2 Theoretical considerations

The history of nonlinear acoustics can be traced back nearly 250 years to Euler's equations for momentum and continuity of motion in a fluid [117]. Since this beginning a great deal of literature has been published by some of the most distinguished scientists in the 18th, 19th & 20th centuries. A detailed historical review of the beginnings of nonlinear acoustics (1755 - 1930) can be found in Hamilton and Blackstock's Nonlinear Acoustics [118]. Following on from this period, several key theories describing the various effects of nonlinear waves under certain conditions emerged. The three most widely accepted and respected equations describing nonlinear propagation were formulated by Burgers, Westervelt and the trio Khokhlov, Zabolotskaya and Kuznetsov (KZK). Each of these individual descriptions of nonlinear propagation all feature a parameter known as the nonlinearity parameter, $\beta$.

5.2.1.1 The Nonlinearity Parameter

The nonlinearity parameter, $\beta$, determines the nonlinear effect on wave propagation. It is defined in Equation 5.1, where the ratio $B/A$ describes nonlinearities in the medium.

$$\beta = \begin{cases} 
1 + \frac{B}{2A} & \text{for liquid} \\
\left(\frac{\gamma + 1}{2}\right) & \text{for gas} 
\end{cases}$$

where \(\gamma\) = ratio of specific heat co-efficient at constant pressure and volume \((c_p/c_v)\)
The quantity $B/A$ is the pivotal aspect of the relation as it has a significant effect on sound speed. It can be expressed in terms of thermodynamic quantities \cite{118} as Equation 5.2.

$$
\frac{B}{A} = 2\rho_o c_o \left( \frac{\partial c}{\partial p} \right)_T + \frac{2\alpha_T c_o T_o}{c_p} \left( \frac{\partial c}{\partial T} \right)_p
$$

where $c_o =$ small signal velocity of sound

$c =$ local velocity of sound in the wave

$\rho_o =$ fluid density

$T_o =$ absolute fluid temperature

$\alpha_T =$ coefficient of thermal expansion

$c_p =$ specific heat capacity at constant pressure

In effect, the quantity $B/A$ will vary depending on the temperature, pressure and density of the medium through which a finite amplitude sound wave is propagating. Examples of this quantity for two common load media are as follows; distilled water has a ratio $B/A = 5$ and air has $B/A = 0.4$, both calculated at a temperature of $20^\circ$C

\textit{5.2.1.2 Westervelt Equation}

The Westervelt equation is an approximation of the second order wave equation and describes the propagation of a nonlinear, quasi-planar sound wave. The derivation of the equation is from that of a thermoviscous fluid perspective incorporating expansions of mass conservation, momentum conservation, entropy balance and thermodynamic state equations. There are certain assumptions embedded within the derivation of the equation, primarily that the propagation medium is homogenous in
composition and its unperturbed density and pressure are uniform. There are others involved but these are more in context within fluid dynamics and are therefore not mentioned in this work. Equation 5.3 represents the Westervelt equation [118].

\[
\left( \nabla^2 \frac{1}{c_o} \frac{\partial^2 p}{\partial t^2} \right) p + \frac{\delta}{c_o^4} \frac{\partial^3 p}{\partial t^3} + \frac{\beta}{\rho_o c_o} \frac{\partial^2 p^2}{\partial t^2} = 0
\]

where

- \( \nabla^2 \) = Laplacian operator
- \( \delta \) = diffusivity of sound (absorption in the medium)
- \( p \) = change in ambient pressure

The expression can be broken down into three unique terms. The first two terms (in parenthesis) represent diffraction and linear propagation and are the wave equation for linear propagation. The third term describes thermoviscous losses in the fluid, or in other words, absorption in the medium. Finally, the last term accounts for the nonlinear effect on wave propagation and is governed by the nonlinearity parameter, \( \beta \).

### 5.2.1.3 Burgers Equation

The Burgers equation is one of the fundamental equations in nonlinear acoustics. It is the simplest model that describes the combined effects of nonlinearity and loss on the propagation of progressive plane waves. It can be derived from the 1D form of the Westervelt equation and is represented in Equation 5.4 [118].

\[
\frac{\partial p}{\partial z} = \frac{\delta}{2c_o^3} \frac{\partial^2 p}{\partial t^2} + \frac{\beta p}{\rho_o c_o^3} \frac{\partial p}{\partial t}
\]
The equation can be split into two parts; the first term on the RHS describes dissipative effects and the second term nonlinear effects. For small amplitude waves the nonlinear parameter can be ignored, $\beta=0$, therefore leaving only dissipative effects left to describe wave evolution. Given this, for a homogenous medium, the propagation of a harmonic wave is described by a solution with exponentially decreasing amplitude [119]. For the case when dissipative effects are negligible, $\delta=0$, only a solution for the nonlinear aspect is available. These particular solutions are shown in Equations 5.5 and 5.6 respectively [118].

$$p = p_o \exp \left[ j \omega \tau - \left( \frac{\delta \omega^2}{2c_o^3} \right) z \right]$$

5.5

$$p = f \left[ \tau + \left( \frac{\beta p_o}{\rho_o c_o^3} \right) z \right]$$

5.6

Equation 5.6 is sometimes referred to as the Earnshaw/Poisson solution to Burgers equation. An in-depth historical review of the Burgers equation is available in Hamilton and Blackstock’s Nonlinear Acoustics [118]. A more general form of the Burgers equation, in spatial representation, can be found in Naugolnykh and Ostrovsky’s nonlinear wave processes in acoustics [119].

5.2.1.4 KZK Equation

The KZK equation is an augmentation of the Burgers equation that accounts for the combined effects of diffraction, absorption and nonlinearity in directional sound beams. Equation 5.7 represents the nonlinear parabolic KZK equation [118].
\[
\frac{\partial^2 p}{\partial z \partial t} = \frac{c_o^2}{2} \left( \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} \right) p + \frac{\delta}{2c_o^3} \frac{\partial^3 p}{\partial t^3} + \frac{\beta}{2\rho_o c_o^3} \frac{\partial^2 p^2}{\partial t^2} \tag{5.7}
\]

The first term on the RHS describes the effects of diffraction where the expression in parenthesis is the transverse Laplacian operator, $\nabla^2_\perp$, which is equivalent to Equation 5.8. The second term describes the effects of dissipation on a travelling wave. Finally, the third term describes nonlinear effects on the evolution of a propagating sound wave.

\[
\nabla^2_\perp = \nabla^2 - \frac{\partial^2}{\partial z^2} \tag{5.8}
\]

The KZK model assumes that the acoustic energy propagates in a fairly narrow beam close to the source axis up to around $20^\circ$ off the z-axis in the far field. Moreover, the approximation is valid for field points that are beyond a few source radii and regions that are not too close to the source plane. The assumption to be satisfied for a circular, piston-like source of radius $a$ and wavenumber $k$, is that $ka \gg 1$. Hence ensuring that the beam is reasonably directional, angular spectrum is narrow and therefore wavefronts are quasi-planar [116, 117 & 120]. The KZK equation is the most widely used model for describing all three effects on the development of a travelling acoustic wave in a homogenous medium, with solutions in the time domain and spectral domain having been developed [121, 122].
5.2.2 Analytical Approach

Due to the KZK model’s ability in describing the evolution of a travelling wave distorted by the effects of diffraction, absorption and nonlinearity within certain limitations, a number of computer algorithms have been proposed to solve the KZK equation numerically. The first approach developed was a frequency domain method, often called the spectral technique, formulated by Aanonsen and co-workers [122]. The KZK can be solved in the frequency domain for monochromatic or tone burst signals by using a finite difference scheme to propagate the wave forward in small steps. Essentially, the pressure wave can be written in terms of Fourier components consisting of the fundamental and its harmonics, truncated at the Nth harmonic to reduce calculation burden, and then substituted into the KZK equation. This enables a set of coupled equations to be derived that allow each harmonic at a grid point in the \((x,y)\) plane at a distance \(z + \Delta z\) to be written in terms of the harmonic amplitudes at \(z\).

The computational procedure for solving these equations can become very time intensive for a distorted waveform involving \(N\) harmonics as the calculation of each harmonic, at each grid point, will involve \(N^2\) multiplications. This problem is further compounded if frequency dependent absorption and strongly focused beams are to be accounted for in the prediction. Furthermore, if source excitation comprises of transients signals, or pulses, then a sufficiently low pulse repetition rate is required to ensure that adjacent pulses in the sequence do not overlap as a result of potential lengthening in duration due to absorption and nonlinearity, otherwise computational burden can be increased by an order of 100.

The spectral approach was followed by a more generic technique in the time domain proposed by Hamilton and Lee [121]. The solving of each term in the KZK model in
the temporal domain significantly eases the strain imposed on computational resources by frequency domain solutions, primarily as the nonlinear term can now be solved with a computational time proportional to $N$ as opposed to $N^2$. In addition, the need to implement Fourier transforms is removed. A marching scheme based on evaluating each of the effects individually was used as the basis for this solution.

5.2.3 Finite Element Approach

Despite the excellent agreement achieved between results from experiment and models based on the KZK equation [120-122], these algorithms remain too idealised for application to more complicated problems involving scattering, cavitation and medium inhomogeneities. A more useful approach modelling non-linear propagation in a load is to implement an incrementally linear solution to the governing elasticity equations [117]. In effect, for transient phenomena this process involves advancing a solution in time by small, discrete steps and modifying material properties according to the new state of the material. These time increments are minute enough that the solution is linear over each step and ensures wave nonlinearities are well-modelled by the incrementally linear approach. This technique, i.e. the constant repetition of simple calculations, is ideally suited to numerical solution using FEM code, specifically, the PZFlex package. The code implements a model based on [48]

$$p = p_o + \rho_o c_o^2 \left( \frac{\rho - \rho_o}{\rho_o} \right) + \frac{B}{2A} \left( \frac{\rho - \rho_o}{\rho_o} \right)^2$$

This allows a selected region of the grid, which is non-piezoelectric in nature, to be allocated material properties that support nonlinear propagation under the correct
conditions, i.e. sufficient pressure amplitudes and/or distance from source. Importantly, this allows nonlinear behaviour to be simulated by FE analysis using PZFlex.

The large majority of literature on nonlinear propagation and simulation techniques focuses on the use of ultrasound in medical applications that operate at relatively high frequencies (2-10MHz). At these frequencies, sufficient pressures for harmonic creation are easy to generate at small input levels, where pressure is proportional to particle displacement (or velocity) and frequency, as such

\[ P = \rho c_\omega \omega \xi \]  

where: \( \xi \) = particle displacement

In addition, the associated reduction in wavelength allows for large propagation distances, in terms of number of wavelengths, over a small range. Therefore, in water and biological samples, it is fairly easy for the evolution of a pressure wave to deviate from its pure sinusoidal form. However, at the much lower frequencies that favour the generation of cavitation (20–100kHz), it is not so easy to generate the pressure levels, or transmission distance, in the load required for the initiation of nonlinear behaviour, with the obvious exception of SONAR. This creates an interesting scenario when investigating the pressure fields generated in the test vessels used prevalently in industrial processing applications, in that it may be possible to stimulate cavitation while the system effectively remains in the linear regime. Indeed, it is not unreasonable to postulate that the generation of harmonics in this situation may be
more of a hindrance when attempting to maximise cavitation events due to the fundamental frequency ‘leaking’ energy to the higher frequencies.

It is often assumed that due to the high input levels associated with ultrasound in many process and ultrasonic cleaning applications, nonlinear waveforms will be readily generated. The presence of harmonics due to nonlinear propagation can have a profound impact on the field profile, causing it to deviate from the linear profile. Therefore, for the accurate characterisation of such acoustic fields it is important that nonlinear effects, if any, can be well-represented. Simulation of these effects in the ultrasonic system described in Section 4.4 is possible through inclusion of a transmission load that supports nonlinear effects in the model. As PZFlex cannot represent piezoelectric nonlinearities caused by high input voltages, any harmonics produced in the load medium would arise due to acoustic activity simulated within the vessel. However, in practice, equipment and device nonlinearities may exist and contribute to the acoustic field. It should be noted that these effects are separate from the nonlinearities associated with the onset of cavitation, both inertial and non-inertial, where bubble oscillations can generate harmonics and sub-harmonics of the fundamental frequency, and where bubble implosions can produce broadband acoustic pulses. These are also not represented directly by PZFlex, although potential simulation methods to emulate cavitation and its affect on the field profile will be discussed later in Chapter 7.
5.3 Linearity Investigation

Preliminary calculations indicate that it would require a considerable input level to generate classical non-linearities in the system and therefore, it is likely that significant cavitational events would occur before this threshold is reached. Subsequently, the generation of bubbles, and other phenomena such as acoustic streaming, would invalidate the measurement technique due to the disruption to the laser light as it traverses the load (Chapter 3). In addition, assuming that the initiation of cavitation within these cells is the primary objective, an increased level of activity will be evident when increased energy levels are introduced at the fundamental frequency. The generation of harmonics due to nonlinear propagation in the transmission load is actually counter-productive to this goal. Energy is transferred from the fundamental frequency, which is more conducive to generating cavitation, to the higher frequency harmonics. However this is based on the assumption that any harmonics are generated as function of the pressure wave in the transmission load.

Aside from potential nonlinearities generated in the load, other sources can contribute to the presence of harmonics and these must be identified prior to defining any conclusions as to the linearity in the load. These sources include; the function generator; high power amplifier; and the transmission devices themselves. Given this, an experimental programme was initiated to evaluate the potential for external sources of nonlinearity.
5.3.1 Drive Equipment

The drive apparatus for the Tonpilz devices consists of an Agilent 33250A arbitrary function generator alone for low voltage tests (10 Vpp), with a Kalmus 155LCR power amplifier included in the arrangement for higher drive input voltages. Both the function generator and power amplifier must be individually evaluated for nonlinear behaviour at relevant frequencies and drive levels. This involves a simple arrangement in which the outputs from both pieces of equipment are connected directly into a spectrum analyser for measurement of any harmonics present at increasing input levels. The main indicator for nonlinear behaviour is typically the magnitude of the 2\textsuperscript{nd} harmonic, with a value above -20dBs considered as significant enough to contribute to the field profile and a value above -40dBs considered worthy of note.

Firstly, the function generator was evaluated for 2\textsuperscript{nd} harmonic in a frequency range from 10kHz to 10MHz. The output of the generator was connected to the HP 8590L Spectrum Analyser and a single frequency at three key voltage levels was recorded; 100mV and 1V cover the operating input frequencies to the power amplifier for high voltage operation and the 10V level represents the input to the system in the absence of amplification. The 2\textsuperscript{nd} harmonic level relative to the fundamental is shown in Figure 5.2. It is clear that in the operating frequency ranges of both Tonpilz devices the maximum 2\textsuperscript{nd} harmonic level is approximately -50dB. Hence, the function generator can be considered linear for the purposes of this work.

Next, the linearity of the 50dB power amplifier is evaluated in conjunction with the function generator. Again, the output of the power amplifier is connected to the
spectrum analyser and harmonic levels recorded from 10 kHz to 10 MHz, with the relative values of the 2^nd and 3^rd recorded in Figure 5.3. From this, it is obvious that the value of the 2^nd does not exceed -40dB throughout the frequency range, and is in fact approximately -48dB and -46dB respectively for the 33 kHz and 40 kHz devices. Using the threshold value of -30dB outlined earlier, it is possible to state that the drive equipment used throughout is linear within the operating frequency and voltage levels employed in this work. Therefore, it can be concluded that any non-linearities found in the system must be attributed to either the transmission device or harmonics generated in the load. This will be investigated in the following Sections.
Figure 5.2. Amplitude of 2nd harmonic relative to fundamental across a range of frequencies for the Agilent function generator at various output levels.

Figure 5.3. Amplitude of 2nd and 3rd harmonics relative to the fundamental across a range of frequencies for the Kalmus power amplifier.
5.3.2 Transmission Device

Commonly with piezoelectric devices, harmonics are present at odd integer multiples of the fundamental thickness mode, where this mode is a half wavelength longitudinal resonance of the device thickness and a function of thickness, hence

\[ f_n = \frac{nV_t}{2d_i} \]

where \( V_t \) is the velocity of sound in the piezoelectric material, \( d_i \) is the thickness, and \( n \) is an integer wavenumber. Given this, \( n \) takes the values 1, 3, 5 etc., this leads to nulls at even integer harmonics. However, if input levels are large enough a degree of nonlinearity can be induced in the piezoelectric material and facilitate the generation of even harmonics. Measurement of these harmonics is a reliable technique to determine the linearity of a device at a given input level.

Device linearity is traditionally evaluated through the detection of harmonics by a hydrophone positioned along the acoustic axis. The ultrasonic device is driven at its electrical resonance frequency either CW or pseudo-CW in water, with steadily increasing voltage levels via a power amplifier and arbitrary signal generator. It should also be noted that maintaining the required free-field conditions for this type of investigation can be problematic due to the reflections from the tank walls.

The signal captured by the hydrophone is transformed into the spectral domain and analysed for the presence of harmonics. In addition, the temporal waveform can be scrutinised for any variances between maximum positive and negative pressure values as this also indicative of nonlinear behaviour. For this investigation, the drive
equipment, characterised in Section 5.3.1, were again used in conjunction with a Bruel & Kjaer 8103 hydrophone. The location of the hydrophone within the field is crucial to the accuracy of the measurement. It should be situated in the far field to ensure it is not in a pressure null, which are common in the near-field. Notwithstanding, it should not be placed in the extremities of the far-field as the increased propagation distance from the source can cause nonlinear behaviour. This makes it difficult to distinguish between harmonics generated from the device and those created through conventional means. Indeed this proved to be the case in practice and an alternative arrangement was devised for such low-frequency devices.

The solution is to simply remove the potential for the transmission load to generate harmonics and evaluate the device in an air channel. As it is extremely difficult to generate conventional nonlinear effects in air, any harmonics detected in the spectral profile must be due to the device. Again, adherence to placing the hydrophone in the far-field must be maintained. This arrangement also negates the difficulties associated with attempting to maintain free-field conditions in a water tank.

An alternative hydrophone better suited to air-coupled ultrasound was employed for the new set-up. A PVDF membrane device with an active area of 5mm in diameter, complete with pre-amplification electronics, was substituted for the Bruel & Kjaer probe. The hydrophone was placed 120mm from the front face (sufficiently in the far field of both devices – see Figures 4.17 & 4.18) for the 33 kHz and 40 kHz Tonpilz transducers. Importantly, the position of the PVDF device remained perpendicular to the acoustic axis to prevent any additional modes being excited in the PVDF substrate, which these devices are particularly susceptible to at lower frequencies.
Each device was excited at its electrical resonance frequency for maximum transmission efficiency with an initial signal of 10 Volts peak-to-peak (with no power amplifier). The amplified signal from the hydrophone is connected directly the oscilloscope, where spectral analysis is instantly available through the FFT function. Figure 5.4 and Figure 5.5 show the spectral content of the received signal from the PVDF membrane hydrophone for the 33 kHz and 40 kHz devices, respectively. From the 33 kHz device, it is clear that a significant 2\textsuperscript{nd} harmonic of -33dB at 64.7 kHz is detected, even at these relatively low input levels in an air environment. This is echoed in the result for the 40 kHz device, where the 2\textsuperscript{nd} harmonic is slightly larger at -28dB for a frequency of 80.2 kHz. An extended FFT for both devices shows higher harmonics are present, up to the 5\textsuperscript{th}, but at reduced amplitudes.

This result demonstrates that these devices are non-linear even at low input signal levels. Indeed, when the power amplifier was utilised with the arrangement to increase input voltage, the amplitude of the 2\textsuperscript{nd} harmonic was seen to increase proportionally with the fundamental component. A search of the literature has failed to produce any previous evidence of this type of behaviour being inherent in Tonpilz devices therefore further investigation of this characteristic was warranted. The laser vibrometer facility provides an excellent opportunity to confirm, and potentially expand upon, the results from the initial hydrophone evaluation.
Figure 5.4. Frequency spectrum of the received signal from the PVDF membrane hydrophone positioned 120mm from the 33 kHz Tonpilz device radiating into an air channel.

Figure 5.5. Frequency spectrum of the received signal from the PVDF membrane hydrophone positioned 120mm from the 40 kHz Tonpilz device radiating into an air channel.
5.3.3 Evaluation by Surface Displacement Measurement

The laser vibrometer efficiently quantifies the surface displacement of a vibrating aperture through the principle of interferometry, outlined in Chapter 3. Through novel application of this established technique, it is possible to analyse the spectral content of an aperture velocity function for any evidence of harmonic content. This is the equivalent measurement that a large-area hydrophone takes, such as the one employed in the Section 5.3.2, as pressure is proportional to particle velocity. Hence, by measuring the particle velocity i.e. the front face velocity, or the front face displacement and applying a FFT, it is possible to confidently state whether a device is linear. This technique was applied to both the 33 kHz and the 40 kHz Tonpilz devices.

There are two measurement options available for applying this technique using the laser vibrometer; a single spot measurement and a complete scan of the vibrating aperture. The latter of the two is the one detailed in this Section due to its completeness and similarity to the measurement taken with the hydrophone in that the entire surface contributes to the pressure field. Given this, the FFT taken is in fact an FFT of the average surface motion for all points recorded by the vibrometer. This provides are more accurate result with a higher signal to noise ratio (SNR) than the single spot inspection.

Once again, the Polytec OFV-3001 laser scanning vibrometer and associated software (Chapter 3) was used for the measurements. Both transducers were driven under the conditions as outlined in Section 5.3.1, with the same drive equipment. It should be noted that the inclusion of the power amplifier for higher drive voltages is possible
with the vibrometer system. The FFT for the 33 kHz and 40 kHz transducers are shown in Figure 5.6 and Figure 5.7, respectively. The 2\textsuperscript{nd} harmonics are clearly evident in both Figures, located at 67.4kHz with an amplitude of -34dB for the 33kHz device, and 80.2kHz with an amplitude of -31dB for the 40kHz device. Importantly, these values correspond well with those obtained through traditional linearity evaluation methods described in Section 5.3.2.

In addition to providing evidence of device nonlinearity through analysis of the spectral content of the front face motion, the laser vibrometer can also provide an insight in the mechanical behaviour at these higher frequencies. From the motion analysis, both the Tonpilz transducers demonstrate reasonably uniform motion across the front face at their respective 2\textsuperscript{nd} harmonics (67.4 kHz and 80.2 kHz respectively). There are several possible reasons for the occurrence of harmonics at such low drive voltages with the Tonpilz devices. The most likely explanation is the construction of the device itself where the individual components collide with each other at the resonance frequency as opposed to simply moving as a complete mechanism. Through this, each impact would generate a series of higher harmonics at multiples of the driving frequency. Another possible explanation is the formation of a standing wave within the centre bolt as it acts like a closed conduit for acoustic energy, similar to the operation of an organ pipe. However, the latter would likely be represented in the detailed finite element analysis conducted in Chapter 4 and this nonlinear behaviour was not evident. It may also be a function of the piezoelectric ceramic operating under intense compression due to the torque of the centre bolt, but again this is unlikely.
Figure 5.6. Frequency spectrum of the average particle velocity for the front face of the 33 kHz Tonpilz device, as measured by the laser vibrometer.

Figure 5.7. Frequency spectrum of the average particle velocity for the front face of the 40 kHz Tonpilz device, as measured by the laser vibrometer.
The presence of a 2nd harmonic at low input voltage levels and single frequency excitation may have an influence on the pressure field generated by the device. In effect, if the contribution of the nonlinear term is significant enough, with the typical threshold being -30dB down from the fundamental, then the true field profile may differ from that predicted by linear models. Chapter 3 discusses the field profile associated with both Tonpilz device and demonstrates excellent agreement between predictions from surface displacement data and FEM generated data. However, both these techniques are fundamentally linear, i.e. single frequency, and are unable to accurately represent the nonlinearities present in the Tonpilz transducers. Therefore, to assess the effects of harmonics on the pressure field it is required to sum the contribution of each frequency through a weighted superposition of each harmonic’s unique pressure profile. For example, utilising FE analysis and being cognisant of harmonic amplitude, creating a series of models where each harmonic becomes the main frequency of interest should allow a reasonable representation of the practical field to be created after summation. Alternatively, a number of frequencies can be extracted from a single simulation, scaled accordingly and summed in post-processing, providing there is suitable bandwidth and the model is adequately meshed. This is also applicable for predictions involving surface displacement data, where magnitude and phase information can be extracted from the laser vibrometer scans and used to simulate the pressure field for each of the desired harmonics, and again summed for a complete field simulation.
5.3.4 Evaluation of Nonlinearities in a Test Cell

Having completed an experimental investigation eliminating any spurious causes for non-linearity that cannot be attributed to the transmission load, it is necessary to evaluate the linearity of both high power systems. Due to the difficulty in inserting a conventional probe into the system and subsequent problems in positional manipulation for accurate spatial alignment, a membrane hydrophone was selected as the most suitable measurement device. This offered the advantage of simply insertion (only a connecting wire need be fed out from the cell) but also reliable spatial positioning as the active element could be placed in the centre of cell through design. The type of device employed is a simple single layer PVDF membrane with a 5mm active element in the centre and an outer diameter equal to that of the cell interior. Section 2.5.2.1 describes this type of device in more detail. Due to the relative nature of the measurements required, i.e. amplitude of 2nd harmonic to fundamental, the device was not calibrated. Furthermore, the hostile nature of measurement environment ensured a short operational life of the membrane due to electrode degradation cause by inertial cavitation. It was not deemed necessary to expend resources (time and expense due to added complexity required) to protect the device from these effects as the experiment did not require long-term use of the hydrophone.

The experimental drive equipment remained consistent with previous measurements. Input voltage was ramped steadily from 50mV to 250mV, at which point the output voltage from the hydrophone exceeded the maximum for accurate scope measurements. Importantly, no pre-amplifier stage was used. The membrane hydrophone was fitted into the cell using thin aluminium holders fixed onto the
aluminium holder ring; these stabilise the position of the device in the centre of the cell with the active element along the acoustic axis.

It is important to note that at the upper voltage range significant levels of cavitation are clearly evident with jetting and sub-harmonics of the bubble collapses visible and audible to the observer. Figure 5.8 shows a picture of the degraded electrode connection after short exposures to the cavitational effects in both systems. Moreover, these effects became apparent at lower input levels of approximately 130mV for the 33 kHz Tonpilz device and 150mV for the 40 kHz Tonpilz. The harmonic levels were monitored throughout with Figure 5.9 & Figure 5.10 showing the relative amplitudes at 250mV input for 33 kHz and 40 kHz systems respectively. It is clear that even at these input levels and in the presence of significant cavitation, the systems can effectively be described as conventionally linear as the 2nd harmonic is -36 dB and -35dB down from the fundamental respectively. Interestingly, these figures are comparable with the harmonic levels recorded in the air channel as a measure of device non-linearity. Indeed, it could be hypothesised that the harmonics witnessed in this instance are a product of the device itself and not the transmission load.
This result would support the initial hypothesis that these systems will begin to cavitate before they develop non-linear propagation effects. Obviously as the transmission distance increases in terms of wavelength this scenario would become less intuitive. However, in the current context it is reasonable to state that linear simulations would reveal an accurate pressure field profile, which in turn would reveal high pressure locations where the likelihood of cavitation occurring is higher. Therefore, measuring the acoustic field profile of such a system operating at low levels would give a representative map of the pressure profile at increased levels when cavitation begins to occur. The following Section will attempt to confirm this result through non-linear field simulation using PZFlex.
Figure 5.9. Harmonic content from membrane hydrophone positioned in cell with the 33 kHz Tonpilz transmitter operating with an input level of 250mV through a 50dB power amplifier

Figure 5.10. Harmonic content from membrane hydrophone positioned in cell with the 40 kHz Tonpilz transmitter operating with an input level of 250mV through a 50dB power amplifier
5.3.5 Nonlinear Simulation in a Vessel

Using the model created in Section 4.4 and implementing a non-linear material as the load medium, it is possible through the mechanisms explained in Section 5.2.3 to evaluate the non-linear propagation effects generated within the load in both high power systems. Each model was swept through increasing input voltage levels from 10V to 500V with pressure histories extracted along the acoustic axis at 2mm samples and at other key off-axis points. Analysis of these pressure histories was then used to determine simulation time to ensure a steady-state condition existed throughout the system. In practice this required a simulation time equivalent to 150 cycles at the main resonance frequency previously determined in Section 4.4, namely 34 kHz and 40 kHz respectively. The initial transient response of the system is then removed to facilitate a more accurate FFT analysis to be completed, with 40 cycles in the steady-state regime isolated for harmonic content. This was implemented across the range of input levels. Figure 5.11 & Figure 5.12 present the frequency spectra across selected input levels for the 33 kHz and 40 kHz systems respectively. For the 33 kHz system, 2nd harmonic presence is initiated at an input of 50V and progressively increases as input voltage reaches 500V. However, the peak value of the non-linear component only just creeps above the -30dB threshold at the highest drive levels. In practice, utilising this level of input is unlikely and counter-productive as the level of cavitation generated would negate the efficient transfer of energy into the load as the bubbles disperse the acoustic energy near the radiating aperture. This is mirrored in the frequency response of the 40 kHz system but with lower 2nd harmonic levels reached in general. This is due to the reduced mechanical displacement produced at the front face in comparison with the 33 kHz Tonpilz, over 30nm/V compared to 15nm/V taken from Figure 4.13 & Figure 4.14 respectively. Crucially, these levels of harmonic
activity are comparable with those measured in Section 5.3.4 where the 40 kHz system demonstrates a lower 2\textsuperscript{nd} harmonic level that the 33 kHz system.

Therefore, through a combination of experimental and simulated results it can be stated that these systems do not generate non-linear propagation effects in the transmission loads at input levels that are large enough to stimulate cavitation effects in the load, where the measurement threshold for non-linearity is taken as -30dB.
Figure 5.11. FFT of pressure at maximum pressure point on the axial plane of the 33 kHz system for increasing voltage input levels.

Figure 5.12. FFT of pressure at maximum pressure point on the axial plane of the 40 kHz system for increasing voltage input levels.
5.4 Conclusions

An intuitive introduction to classical nonlinear propagation with reference to high-power cells has been described in this Chapter. The hypothesis proposed contradicts the common held perception that these systems are highly non-linear while cavitating and therefore, linear models for predicting regions of high pressure are invalid. A review of the common non-linear models is presented, each accompanied with a brief synopsis of the merits. The most comprehensive analytical non-linear model is a time or frequency domain solution to the KZK although it is deficient when representing focused sources and medium in-homogeneities. Given this, a numerical solution to the problem was considered using transient FEA software capable of modelling nonlinear propagation. The FEA code allows for small incremental linear solutions to be culminated over time, providing an accurate solution to non-linear problems. The PZFlex code was utilised to investigate the non-linear behaviour associated with the low frequency Tonpilz devices in a high power setting.

Spectral analysis was performed on associated equipment in order to isolate any effects separate from those occurring in the transmission load. Equipment nonlinearity was found to be nominal while 2nd harmonic levels in the Tonpilz devices were approximately -30dB relative to the fundamental amplitude, even at low input levels. Next, a membrane hydrophone was employed to measure nonlinear effects present within the vessel load. Taking into account the intrinsic non-linearity of the tonpilz devices, the conventional non-linear behaviour, i.e. steepening of the wave profile, was found to be minimal even when the system was observed to be cavitating. Linearity at these input levels was confirmed via transient FE analysis.
Therefore, it is reasonable to state that these systems are inherently linear in conventional terms, even when the cavitation threshold is surpassed. This facilitates the use of reliable linear models to predict regions of high pressure, and where cavitation is likely to occur, in such systems. Consequently, any measurement technique applied at low input levels will accurately represent the field profile until cavitation begins. The next Chapter discusses the implementation and results from a novel measurement technique that operates at sub-cavitational power levels.
CHAPTER 6

6. MEASUREMENT OF ULTRASONIC FIELDS IN SEALED VESSELS
6.1 Introduction

Previous Chapters have outlined a viable non-invasive measurement technique for the inspection of pressure fields within cylindrical vessels and provided detailed FE analysis of low frequency, high power devices for use in simulations with these vessels. The measurement method has proved effective in mapping airborne pressure fields from a 33kHz Tonpilz device under free-field conditions in a shielded environment, with good agreement between experiment and theory achieved (Chapter 3). Additionally, the FE representation of the Tonpilz devices (Chapter 4) shows excellent corroboration with both electrical impedance and laser vibrometry measurements. Hence, there exists a measurement system and a theoretical framework for the quantification and simulation of internal pressures. This combination offers the potential to mutually confirm both the experimental technique and the viability of FE methods as a virtual prototyping tool.

This Chapter assesses the veracity of both the non-invasive measurement method and the detailed finite element simulation of cell pressure dynamics. Two different systems will be evaluated; one cell featuring the 33 kHz Tonpilz, and the other using the 40 kHz Tonpilz with both operating into a water load. Following this, reasons for possible discrepancies between theory and experiment will be listed in addition to a discussion on some of the limitations of the scanning procedure. Finally, a selection of scenarios varying device arrangement, frequency content and stirrer presence will be theoretically investigated for their influence of the internal pressure field, and the potential to use these influences to an advantage in a practical high power reactor vessel will be presented. It is important to note that all the FE predictions of pressure
in the following sections have the capacity to represent non-linear propagation (Chapter 5).
6.2 33 kHz Tonpilz System

Previously, Chapter 3 described the validation of the angular and positional manipulation of the laser for use in cylindrical vessels. Following this, a simple cylindrical perspex test cell incorporating the 33 kHz Tonpilz device was constructed for the measurement of standing wave patterns generated within these structures. The 33 kHz Tonpilz was fitted into a perspex holder that in turn was fixed to the open end of a perspex cylinder sealed with a PVC lid at the opposite end, review Section 4.4 for details. The cell consisted of a 100 mm outer diameter, with 5 mm thick walls, and a total length incorporating both the device and the PVC lid of 169 mm. The refractive index values of the Perspex and water are 1.49 and 1.33, respectively [123]. These dimensions were specifically chosen to ensure the standing wave pattern between the device front face and the reflecting lid falls within resonant criteria (an odd number of quarter wavelengths) around the optimal transmission frequency of 33.7 kHz, according to Equation 6.1. Furthermore, distilled water at room temperature was used as the transmission medium in an attempt to minimise any air bubbles present both on the front face, possibly reducing the energy coupling into the load, and within the measurement area to prevent the scattering of the interferometer beam.

\[ H = (2n + 1) \frac{\lambda}{4} \]  

6.1

where

- \( H \) = Height of water column in reactor
- \( n \) = integer number
- \( \lambda \) = wavelength of sound in load media.
A section of the water column 65 mm in length and 56 mm in diameter was examined taking 26 slices, sampled at 2.5 mm along the acoustic axis. Each complete slice consisted of 18 projections containing individual samples 2 mm apart (<\(\lambda/4\)) in the lateral dimension; correction for the refractional effects of the cell was applied. The transducer was driven at nominal power levels, 10 volts peak-to-peak, in order to avoid cavitation from occurring, and at excitation frequency of 34 kHz (Section 4.4). Despite the fact that non-linear effects can be experimentally detected and accurately modelled in the FE environment, the absence of notable harmonics in the measured data confirmed the linear operation of the system at these nominal power levels. The duration of each complete scan was approximately 16 hours. The measured data were filtered to remove any high frequency noise inherent in the reconstruction process using a 3x3 median filter [124], and linearly interpolated for clarity. A 2D axisymmetric FE model was used to generate images of pressure along the centre plane in the z-direction for comparison with measured data. Although a more complete 3D model is feasible, the accuracy of the axisymmetric grid in PZFlex is more than sufficient and hence, the exponentially increased processing time that would be required for the 3D model is not warranted in this instance.

The normalised reconstructed and FE generated harmonic pressure fields through the centre of the cell are shown in Figure 6.1(a) and (b) respectively. It is evident that two main regions of intensity are present, separated by around a half wavelength (approximately 23 mm) from each other. It is also clear that following a null in intensity at around 60 mm, a further peak is present prior to the water/PVC boundary (RHS of figures). The close correspondence between theoretical and measured pressure is obvious from inspection of Figure 6.1(a) and (b), however some
discrepancies do exist. To appreciate these differences more clearly, the pressure profile along the central transducer axis for both the predicted and measured intensity fields is presented in Figure 6.1(c). These subtle differences between the two profiles are considered to arise from experimental alignment inaccuracies in the measurement of the sound field in the cell. The initial starting position of the scan assumes that the position of the laser beam is exactly through the centre of the cell at normal incidence to the cell wall, and that the sensor head is perpendicular to the acoustic beam axis. It is also assumed that the cell itself is a perfect cylinder with no deviations in wall thickness around the circumference, and there are no structural faults in the Perspex that would effect the transmission of the laser light through the cell. However, measurement with a micrometer established that not only does cell wall thickness vary locally around the cylinder (± 5%); slight imperfections (scratches and small smudges) in the Perspex exist at arbitrary points. Therefore, some of the pressure measurements will have been compromised to varying extents during the scanning procedure. In addition, exact positioning of the sensor head at the initiation of a scan is subject to a positional error of approximately ±0.5 mm. Moreover, measurement errors due to the anomalies present in the cell geometry are increased somewhat when refraction correction is taken into consideration: incident and transmission angles of the laser light will differ from the ideal values generated by the algorithm. Finally, the field from the FE model is predicted under ideal conditions with the operating device displaying perfect symmetry around the acoustic axis. Laser vibrometry has previously shown the mechanical behaviour of the Tonpilz device is slightly asymmetric about this axis (Section 3.4).
Nevertheless, there is good agreement between the measured standing wave intensity profile and that generated from the FE model and indicates that the internal pressure field for such ultrasonic systems can be accurately predicted using the PZFlex package.
Figure 6.1. 33 kHz Tonpilz sealed in the 169 mm cell: (a) Experimentally measured internal pressure in the central plane along the acoustic axis (normalised). (b) FEM predicted pressure in the central plane along the acoustic axis (normalised). (c) Comparison of normalised axial pressure profiles from the measured acoustic field and that predicted by FEM.
6.3 40 kHz Tonpilz System

For a more thorough validation of the measurement method another system was considered for investigation; a 40 kHz Tonpilz device was fitted into an alternative perspex holder which was subsequently fixed onto the open end of a perspex cylinder in the same manner as described in Section 6.2. The dimensions of the cell remained consistent with the previous arrangement, as did all other experimental conditions, with the exception of a reduction in the cell length by approximately 40 mm, giving a new total length of 129 mm including the transducer and holder. The refractive indices of the cell and load materials remained the same. Contrary to the cell used in conjunction with the 33 kHz device, the dimensions of this cell were not chosen to fulfil any criteria such as the creation of resonant standing waves. This was done in an effort to remove some of the predictability of the field profile and further verify both the modelling and the measurement technique.

The excitation frequency of the device was chosen to remain at 40 kHz in accordance with the results obtained in Section 4.4. It should again be noted that these types of device are very sensitive to the loading parameters they are exposed to in practical applications and are known to deviate significantly from the unloaded condition. This can have effects on both the electrical impedance, as demonstrated, but also on the surface displacement profiles of the device which may deviate from the measured piston mimic that was shown in Chapter 3.

The reduced length of the new cell results in a section of the water column, 40 mm in length, being examined taking 20 slices, each 2 mm apart, along the acoustic axis. The
parameters for each scan, along with any filtering and interpolation employed, remained consistent with those established in the previous example (Section 6.2). A 2D axisymmetric FE model incorporating the new holder and cell dimensions was developed in order to provide a suitable comparison for the measured field profile. The normalised reconstructed and FE generated field profiles are shown in Figure 6.2 (a) and (b) respectively. From inspection of the two profiles it is clear that several similarities exist, specifically, the two main intensity peaks at around 15 and 38 mm along the central plane, with the latter being more evident. The images also display similar beam widths around these regions, although again, the second region demonstrates more correspondence between theoretical and measured profiles. However, the intensity null predicted by the FE model at around 22 mm, is not well represented in the measured profile. There may be several reasons for this discrepancy, outlined in the previous section, with the ‘perfect world’ representation of the system by the FE code being the principal. Additionally, there may be external loading factors to consider which are not included in the model for the sake of brevity, such as the influence of the aluminium holder the entire unit is fixed into during the scanning procedure, which will be discussed in Section 6.4. The pressure profile along the central axis, shown in Figure 6.2 (c), illustrates both the similarities and the differences between the experimental and theoretical fields more clearly. Importantly, this scan was subject to the same positional errors as the previous 33 kHz device.
Figure 6.2. 40 kHz Tonpilz sealed in the 129 mm cell: (a) Experimentally measured internal pressure in the central plane along the acoustic axis (normalised). (b) FEM predicted pressure in the central plane along the acoustic axis (normalised). (c) Comparison of normalised axial pressure profiles from the measured acoustic field and that predicted by FEM.
6.4 Modal Spreading

Aside from the main operational modes of the active device, in this case the longitudinal and flexural modes of both Tonpilz devices, the cell itself will also demonstrate distinct vibrational modes depending on its geometry, material properties and loading conditions. Figure 6.3 (a) illustrates an example of the discrete modes that may exist for a typical undamped system incorporating a cylindrical cell. If no energy is lost in the system the energy within the cell is representative of one specific frequency only, hence, distinct pressure nodes and anti-nodes will be evident along the acoustic axis. However in reality, a degree of damping will occur and this energy loss tends to broaden the frequency spread of each mode, shown in Figure 6.3 (b). Therefore, the greater the damping: the greater, typically, the spread in energy (Figure 6.3 (c)). In addition, these modes are very sensitive to any changes in system dynamics, in particular operational frequency, which will be discussed further in Section 6.5.1

This leads to the following effect; as the frequency increases, the number of modes around a given frequency band increases, resulting in overlapping to an increasing extent. This increase in modal density will eventually give the appearance of a diffuse sound field within the system, where the energy from adjacent modes spread into one another removing the individuality of each, as depicted in Figure 6.3 (d). The critical frequency for this to occur is the Schroeder frequency. The value of this frequency depends on the damping; the higher the damping, the lower the Schroeder frequency. This relates to the comparison of experimental and theoretical profiles in that the FE representation of the system has no damping imposed on it i.e. it can be assumed to be
a free-standing cell with perfectly reflecting boundaries. However, the experimental set-up has definite damping conditions applied to it as the PVC lid is fitted into an aluminium holder that is part of the rotational plate, and hence, the entire scanning frame. Obviously it is highly impractical to model the entire environment surrounding the ultrasonic system in the FE realm, therefore this damping effect, and subsequent modal spreading of energy, is not theoretically represented.

There are several possible solutions to this problem using PZFlex. The first is the application of absorbing boundaries at the PVC lid. This option allows the mimicking of a perfect absorber at this boundary, where, ideally, no reflected energy would be transmitted back into the system. However, mimicking the behaviour of a perfect absorber is not an easy process and results in the incomplete absorption of many non-normal waves incident upon the boundary. This causes an unstable reverberation to occur within the system resulting in the oscillation of pressure toward infinity.

A possible alternative solution is to artificially increase the damping in the material properties of the constituent materials within the model. Nevertheless, this approach would require a trial and error approach to accurately represent any external damping and the level of accuracy obtained in the final solution will likely not warrant the resources allocated to obtain it.

Finally, a simplistic approach is to individually evaluate the internal field at several frequencies of note and superimpose these fields in an attempt to emulate the spreading of acoustic energy. This solution is not advised for accurate results but does
demonstrate the degeneration of the clean modal patterns associated with a single frequency.

![Graphs showing frequency vs. power for different scenarios.](image)

Figure 6.3. (a) Ideal scenario with each mode occupying a discrete frequency (b) slight modal spreading due to damping (c) increased damping leading to greater spread in frequency (d) all but the lowest frequency mode are lost due to adjacent modes diffusing into one another.

### 6.4.1 Harmonic influence

As discussed in Chapter 5, PZFlex is able to effectively simulate nonlinear propagation in the transmission load if source amplitude and/or transmission distance is large enough. However, the code cannot represent device nonlinearities similar to those discovered with the Tonpilz devices (Chapter 5) where there exists a sizeable second harmonic component when operating in CW mode. This presents a problem when extracting the pressure field generated by such devices as it cannot be represented completely due to the potential exclusion of a source contribution.
Although not an ideal solution, it may be possible to estimate the effect of such a harmonic by defining reasonable threshold for harmonic amplitude, calculating the acoustic profile generated by it and adding it to the field of the fundamental. The main drawback of such a method is that attempting to represent a nonlinear process through superposition of linear components is unlikely to yield a suitably accurate result, although it may account for some of the discrepancies between the measured and simulated profiles. However, harmonic levels of -34dB and -31dB for the 33 kHz and 40 kHz devices respectively at current drive levels suggests that this factor will not have a significant impact on the field profile and can subsequently be ignored. Upon increasing source amplitude, while operating in air, it was found the level of second harmonic did not increase beyond these values, hence, in a practical application, this factor can again be assumed to have minimal impact on the system acoustic field profile.

6.4.2 Non-Modified Tomography

In Section 3.7.1 it was mentioned that near the normal incidence to the cell, the effects of refraction would be minimal and that samples taken in this region would remain valid in the reconstruction procedure. In an attempt to illustrate this principle and the improvement angular correction makes to the measured profile, the algorithm was removed from the scanning routine to leave basic straight-line tomography as the measurement method. The field produced by the 33kHz Tonpilz device, under the same conditions as presented in Section 6.2 was measured in this manner and is shown in Figure 6.4(a). It is clear from this image that the breakdown in parallel projections results in the erroneous measurement of the pressure field within the cell. Notwithstanding, from inspection of the axial profiles of the FEM, modified
tomography and non-modified tomography, demonstrated in Figure 6.4(b), the discrepancies between the three are not so easily recognisable. In fact, the non-modified axial profile shows reasonable correspondence to both the FE profile and the measured profile using the geometrical algorithm to correct for refraction, with some minor deviations. This supports the earlier hypothesis that quasi-parallel projections taken near the centre of the cell will result in reasonably accurate reconstructions of pressure at these regions (Section 3.7).

The result in Figure 6.4 illustrates two important points; firstly, that for an accurate 2D profile to be gained, the geometric correction for refraction is a crucial aspect of the inspection procedure; secondly, that it may be possible to achieve a fairly accurate axial pressure profile without recourse to the complicated synchronisation factors involved with manipulating laser firing angle. In this case, axial only data would be sufficient if a rough estimate of pressure node position was required. However, in most cases a complete 2D profile along the acoustic axis would be necessary, and this facilitates the need for a complete 3D data set of pressure values.
Figure 6.4. (a) Experimentally measured internal pressure from the 33 kHz device in the central plane along the acoustic axis – without any correction for refraction (normalised). (b) Axial profiles comparing measured pressure: with correction, without correction and FE generated for 33 kHz device.
6.5 System Modifications

In this Section the dynamics of the 33 kHz system will be altered in several ways, both experimentally and theoretically, in an attempt to assess the impact that drive variations may have on the system performance. Firstly, the operational frequency of the driving transducer will be moved slightly off-resonance and the pressure profile re-measured and compared with FE predictions at the new frequency. Next, a simple stirrer mechanism is simulated within the vessel with its effect on the pressure profile investigated for a variety of positions. This concept is then extended to simulating an alternative stirrer mechanism that may enhance system performance. Finally, an atypical approach to creating additional regions of high pressure within the cell through two transducers operating at different frequencies is simulated and presented.

6.5.1 Drive Frequency Variation

This is the simplest form of system alteration and it is done to illustrate the sensitivity of the internal pressure profiles to small changes in acoustic wavelength. It is assumed in this case that wavelength is the only major change as the Tonpilz transducer will continue to radiate as a piston, albeit at a slightly reduced output, and therefore any alterations in the profile are due to changes in cell dynamics due to wavelength alterations. Importantly, all other experimental and simulation conditions are consistent with those described in Section 6.2.

Figure 6.5 (a) shows the measured pressure profile in the 33 kHz system for a drive input of 34.5kHz, 200Hz below the electrical resonance. From the Figure, it is clear that this profile deviates slightly from those at 34.7kHz in Figure 6.1, in particular
with the first pressure peak decreasing in amplitude. This is also represented in the simulated profile at this operating frequency, shown in Figure 6.5 (b), which shows good correspondence with the experimental measurements. The subtleties between the two profiles are more clearly shown in the axial pressure profile in Figure 6.5 (c).

This result highlights the requirement for an understanding of the potential variations that are possible when operating conditions move from the ideal. For example, in an ultrasonic cleaning system, these variations in field profile may cause a reduction in performance due to the decreased pressure at a particular location. This eventuality serves to reinforce the concept that reliable spatial knowledge of the internal pressures within such systems can lead to a more efficient system. Moreover, the close correlation between FE and experimental fields further establishes confidence in the use of accurate FE models to predict the pressure profiles, without recourse to complicated measurement procedures.
Figure 6.5. 33 kHz Tonpilz sealed in the 169 mm cell and driven at 34.5kHz (off resonance): (a) Experimentally measured internal pressure in the central plane along the acoustic axis (normalised). (b) FEM predicted pressure in the central plane along the acoustic axis (normalised). (c) Comparison of normalised axial pressure profiles from the measured acoustic field and that predicted by FEM.
6.5.2 Stirrer Position and Design

At present, this work has concentrated on the evaluation of pressure fields within simple cylindrical vessels with planar reflectors (transducer front face and cell lid) at opposite ends. However, in practice, the majority of such systems will also incorporate a stirrer mechanism of some description at some position within the fluid load. As has been demonstrated in Chapter 4, the presence of a measurement probe in the vessel can have a profound impact on the pressure profile, and the influence of a mechanical stirrer is no different.

In this Section, a simplistic representation of an aluminium stirrer is modelled at various positions in the load. Once more, the position is restricted to the centre of the model due to the axisymmetric conditions that exist. Here, two main positions were chosen to highlight the effect the stirrer mechanism may have on the acoustic profile, 29mm and 36mm from the 33 kHz Tonpilz front face. Figure 6.6 illustrates the field within the cell for both these positions. The transducer was driven CW at 10Vpp until steady state pressure was achieved, in accordance with previous simulations. From the Figure it is clear that by simply adjusting the stirrer position by 7mm a significant increase in intensity at several points in the load can be achieved. Incidentally, this is similar to the effects noted when hydrophone position was investigated earlier in Chapter 4. As an addendum, it may be possible to reduce the overall effect of the stirrer mechanism through choosing a material that has an acoustic impedance similar, or close, to that of water. This would minimise the reflections at the load/stirrer boundaries, but it may also serve to reduce the overall energy in the system as a larger quantity of energy would be transmitted into the stirrer material.
Figure 6.6. Theoretical investigation of influence of stirrer position on the field profile within a cylindrical test cell. (a) Stirrer positioned 29mm from front face and (b) stirrer positioned 36mm from front face. Both plots are normalised.
It was considered appropriate to investigate the stirrer’s design and position in the cell for the enhancement of system performance. An alternative mechanical design was investigated through simulation. In this case, the main assumption is that a pressure doubling effect can be achieved at the fluid/solid interface due to the transmission and reflection co-efficients associated with such boundaries. With this in mind, a stirrer with a larger surface area was placed in the load, again in the centre, and the pressure profile extracted for optimum transmission frequency in the 33 kHz system. Aluminium was again used as the stirrer material. It is important to note that all pressure profiles in Figure 6.6 and Figure 6.7 are normalised to the maximum pressure in Figure 6.6 for ease of comparison. Also, the drive conditions remained the same.

Several advantages can be elicited from this type of design. Firstly, the potential to achieve a more thorough mix is possible due to the increased surface area present. Secondly, the stirrer can be tuned to create localised sites of cavitation and hence the user can target particular areas of load towards these regions for increased reaction efficiency.
6.5.3 Dual-Frequency Resonator

As an alternative to the single transducer systems featured in this work, there exists the potential to multiplex several transducers operating into the same load. Such multi-device systems offer the opportunity of maximising intensity within the transmission load while minimising the input across each individual transducer. Expanding on the concept of multiple devices, incorporating transducers operating at different frequencies may present several performance advantages over a single frequency system. For example, choosing the electrical resonances of both Tonpilz devices as the input, 33.7 kHz and 40.1 kHz respectively, it is possible to create a ‘beat’ frequency equal to the difference between the two, 6.7 kHz, which may aid the onset of cavitation.

Figure 6.7 Field pressure in a sealed cell for an alternative stirrer design in conjunction with the 33 kHz Tonpilz transducer.
Owing to the complex nature of the Tonpilz FE models, it is highly impractical to incorporate more than a solitary device into a single model, and hence a simpler approach was adopted. It is known that both devices demonstrate pseudo-piston behaviour at the resonance frequencies, therefore, the model can be simplified to a cylindrical perspex cavity with two sinusoidal CW pressure loads applied to the opposite planar surfaces of the cavity. In this example, the RHS represents the 40 kHz device and the LHS represents the 33 kHz device. Importantly, the cell dimensions have been altered to attempt to approximate a quarter wavelength resonator that is a compromise for both wavelengths as opposed to optimised for a single operating frequency. For comparison, the same cell is insonified with two 33 kHz and two 40 kHz transducers respectively, with the best performing arrangement chosen for comparison against the dual-frequency reactor. In this case, two 33kHz transducers out-performed the 40kHz devices. Figure 6.8 (a) shows the field profile from two 33 kHz transducers operating together, while Figure 6.8 (b) illustrates the improvement that can be achieved via a dual-frequency system, echoed in the axial profile (Figure 6.8 (c)), through greatly increased pressure generated in the load.
Figure 6.8. Simulation of two transducers acting as a pressure load on opposite ends of a cell: (a) FEM predicted internal pressure in the central plane along the acoustic axis (normalised) from two 33 kHz Tonpilz transducers. (b) FEM predicted pressure in the central plane along the acoustic axis (normalised) from a 33 kHz (bottom) and a 40 kHz (Top). (c) Comparison of normalised axial pressure profiles from single and dual frequency systems.
6.6 Summary

This Chapter has described the investigation of a non-invasive pressure measurement system for the evaluation of pressure fields generated by low-frequency ultrasonic transducers operating within sealed cylindrical vessels. Two different systems, 33 kHz and 40 kHz, were employed to verify the technique with measurements and corresponding finite element simulation results presented for each. Indeed, it has been established that good agreement is found between theoretical predictions for internal pressure fields and those recorded experimentally. This is encouraging as the level of accuracy demonstrated in this work is greater than similar attempts with measurements made through probe insertion [32]. Furthermore, confirmation of the FE model’s ability to represent the internal field develops the potential of designing larger scale vessels as, initially, a purely theoretical exercise, i.e. virtual prototyping and performance assessment through simulation. Through this, the resource-intensive iterative construction approach, normally employed for vessel design, can be negated to a large extent and overall cost reduced. Moreover, it also offers the potential for ‘blue sky’ ideas to be pioneered, with cost-effective investigations into alternative cell geometries, transducer configurations, material types, harmonic suppression etc. becoming possible without recourse to complicated prototype fabrication processes. Section 6.5 demonstrates some possible modifications to the simple high power system, with the aim of enhancing regions of high pressure and, therefore, increasing the likelihood of inducing localised inertial cavitation within these zones.

Several limitations of the measurement technique have also been discussed with their effect on measurement accuracy highlighted. Due to the low operational frequencies
and relatively large wavelengths, slight discrepancies in cell geometry, material inhomogeneities and positional errors during the scan process have minimal effect on the final reconstructed field profile. In addition, it has been described why the technique can only interrogate a limited region of the internal load due to the effects of refraction at the cell walls. However, as the areas of interest exist well within accessible ranges, i.e. through the centre of the cell, this is not seen as a major inadequacy of the measurement technique.
CHAPTER 7

7. CONCLUSIONS AND FUTURE WORK
7.1 Conclusions

7.1.1 General Overview

Low frequency, high power ultrasound has become an integral component in many industrial processing applications. The unique and dramatic effects of inertial cavitation generated at high pressure regions, often serves as the primary mechanism for increasing reaction speed, eliciting particular chemical effects and for removing surface contaminants on conventionally hard-to-clean objects. Moreover, high power ultrasound is now being considered as a viable means for cheap water purification and other similar socially beneficial applications.

However, with the increased level of activity comes the increased need for accurate field characterisation and measurement techniques. This is particularly true when attempting to predict the regions where cavitation is likely to occur. Unfortunately, due to the reverberant nature of many applications, coupled with the adverse conditions presented by cavitation, contemporary measurement techniques can offer an inaccurate representation of the pressure profile and also suffer irreparable damage. The physicality of the probes and the difficulty in scanning them throughout the field often compromises the data captured. As a result, the verification of any simulation data is notoriously difficult to achieve. Consequently, this then negates the use of field simulation packages as an effective design tool for the optimisation of reactor design and more empirical, resource-intensive methods are typically recruited. Therefore, this has necessitated the requirement for a non-invasive pressure measurement system capable of mapping the internal fields of such reactors. Chapter 3 of this Thesis presents such a method incorporating laser interferometry and modified tomographic
scanning routines. In addition, an extensive Finite Element analysis of two typical low frequency high power transmitters utilised with cylindrical reactors cells, has spawned a method for reliable simulation of internal field pressures. Chapter 4 describes the modelling in detail and also presents predicted field profiles for said fields when conventional measurement probes are inserted, with variances from the normal evident.

In Chapter 5 an intuitive investigation is presented into potential non-linear propagation in the transmission load and the affects it may have on the field profile. Interestingly, and contrary to commonly held notions, it was found that these systems are not exclusively non-linear and, indeed, they appear to operate in the linear regime even upon the initiation of cavitation. This allows the possibility of using linear simulation methods as a means to predict pressure profiles in reactor designs. Finally, a complete verification of the measurement method is presented in Chapter 6 where good agreement with simulated results is achieved.

### 7.1.2 Non-Invasive Pressure Measurement

A novel non-invasive pressure measurement technique capable of mapping 3D pressure fields within sealed vessels was developed to solve the issues that are generated when conventional probes are utilised for field profiling. The technique is based on the acousto-optic effect which facilitates acoustic pressure measurement through quantifying phase modulations induced in coherent light beams. Utilising laser interferometry as the light source, this method offers the user an opportunity to quantify average pressure through phase changes induced in a beam of light traversing the acoustic field. In order to achieve a complete 3D map, simple tomographic
scanning routines and reconstruction algorithms are employed. This technique is then successfully demonstrated with application to a 33 kHz Tonpilz transducer operating in an air transmission channel.

The Tonpilz transducer was driven in a non-reverberant air load at its resonant frequency with a nominal input voltage of 10Vpp. The front face displacement was captured using laser vibrometry and used as input to an accurate field prediction model for homogenous media. Next, a 2D pressure slice at a region 95mm from the front face was interrogated using the non-invasive technique and reconstructed with the FBP algorithm. Subsequent comparison with field simulation revealed good correlation, particularly considering air-coupled pressure measurements are often subject to inconsistencies due to environmental conditions, e.g. ambient temperature and pressure variations over time.

This technique was then modified to account for the refractional effects caused by the laser light interacting with the cell walls at non-normal angles of incidence. The best solution was to simply modify the laser firing angle through the inclusion of a precise controllable rotary motor. A simple geometric ray-tracing algorithm was conceived which calculated the desired firing angle. This was then incorporated into the automated scanning procedure applicable for a generic load media and test cell design. Therefore, there exists a novel technique for the accurate non-invasive measurement of pressure fields both in reverberant environments and in normal free-field conditions. The technique is inherently wideband and sensitive while demonstrating the potential for high spatial resolution.
7.1.3 Pressure Field Simulation for High Power Systems

Extensive Finite Element Analysis was completed of both the Tonpilz devices alone and as part of a high power system in order to provide a reliable comparative metric for experimental work. Electrical impedance profiles, surface displacement and free-field pressure fields were used as benchmarks for the modelling process during the transducer characterisation stage and continual referral to experimental results for these properties ensured an accurate theoretical foundation was laid. From this, a 2D representation of the high power systems was formed and used as the basis for simulating pressure fields in an enclosed area. This was then employed to evaluate the potential effect probe insertion would have on the complex standing wave profiles generated in said systems. A commercially available low-frequency piezoelectric probe and a custom-made PVDF membrane hydrophone were both simulated in a selection of planes along the acoustic axis and compared with an undisturbed profile. As expected the PVDF probe elicits less impact on the overall profile due to the material properties being close to that of water. Indeed, this led to the postulation that a 2D, or even a 3D, PVDF array could be used for rapid pressure field characterisation in practical applications. This excluded, it has been reliably established that FEA using the PZFlex code can be used as a reliable means of simulating the behaviour of high power systems and as a design tool for the rapid prototyping of new concepts.
7.1.4 Quantifying and Simulating Non-linear Effects in High Power Systems

As it is typically assumed that such high power systems generate significant non-linear effects in the load media, and that these effects have a profound impact on the acoustic pressure profile, an investigation to assess their impact was completed. After measuring the harmonic content of the drive equipment, it was found that both Tonpilz transducers are non-linear even at very low operational levels, generating a 2nd harmonic value approximately -35dB down from the fundamental at input of 10Vpp while operating in air. Interestingly, when operating in the high power systems the non-linear effects detected by a PVDF hydrophone in the load were minimal even at input levels sufficient to generate noticeable cavitation. The extensive modelling procedure employed for field prediction was then extended to incorporate possible non-linear effects generated in the transmission load in both high power systems. A series of simulations confirmed that both systems operate primarily in the linear regime at input voltages approaching 100Vpp. Therefore, it can be stated that when attempting to predict the pressure field profiles within such systems, linear simulations at low input levels can be used as a reliable tool providing that sub-cavitational conditions are maintained throughout.
7.2 Suggestions for Future Work

7.2.1 HIFU and Processing

Future application of this work within the context of HIFU systems is two-fold; first, the non-invasive technique has potential to be extended to measure non-linear fields from focussed devices; second, this work forms a platform from which novel research into HIFU techniques for processing enhancement can be launched.

As discussed in Chapter 2, accurate measurement of HIFU fields is subject to problems such as damage to the measurement probe, inadequate spatial resolution, bandwidth limitations, sensitivity and probe invasiveness. All this can lead to the inaccurate characterisation of HIFU devices, a precarious situation when dealing with patient safety and acceptable energy levels. There is also the real danger that incorrect focus prediction may incur damage to healthy tissue near the target region. Remaining cognisant of these facts, the non-invasive measurement technique described in this thesis is ideally placed to act as a field characterisation tool for many HIFU devices. Measurement through laser interferometry offers the potential for wideband measurements with little or no variation in sensitivity. In addition, this bandwidth makes the technique intrinsically capable of non-linear pressure detection, where harmonic behaviour can be accurately captured. Furthermore, the utilisation of micro-positioning stages and small laser spot size provides the opportunity for high spatial resolution measurements. The prohibitively long scan time that is associated with such fine measurements can be reduced if the device is axisymmetric around the acoustic axis, and hence, only one projection need be taken for reconstruction. Implementing such a measurement method in conjunction with CUE’s expertise in
transducer prototyping and characterisation would notably advance the development of contemporary HIFU devices, with the possibility of new innovative designs being realised.

The use of HIFU techniques within the processing arena is a completely novel research application according to the literature. The objective in this work would be to optimise the processing systems in the pharmaceutical and food industries through building on the FE design methods pioneered in this work, and combining them with phased array and multiple source technologies. The use of a 2D phased array within a processing system provides the capability to create focussed high pressure regions with in a 3D volumetric space. This is of particular use if implemented as part of a flow based system. Moreover, this work can be extended to include multiple devices where FE modelling methods would provide a valuable insight into the complex field profiles generated. Finally, this work would be complemented by current process-monitoring research ongoing at Strathclyde. Indeed, such is the potential of this future research a recent funding proposal for £380k has been granted by the EPSRC for this investigation.

### 7.2.2 Scaling for Industrial Applications

The scale-up of sonochemical reactors necessary for most applications has always been one of the limiting factors in applying such technologies in industry. What works well on a laboratory scale may indeed be useless on a much larger industrial scale. This shortfall is particularly acute when dealing with applications such as water and sewage treatment, as discussed in Chapter 2. However, as an alternative solution, it may not be necessary to scale ‘up’ to suit an application, rather scale ‘out’ with the
implementation of many smaller, laboratory scale vessels replacing one large container. This is an interesting possibility within the context of this work as the results in this thesis would be directly applicable to industrial situations. Further, by utilising many smaller reactors it is possible to apply tighter feedback control systems to the complete process e.g. more reliable temperature control and flow rate. This would act as an additional method of system optimisation providing adequate process monitoring systems are in place, of which CUE has several current programmes of work running in conjunction with the Chemistry department.

In terms of system modelling, one reactor design can be used as the generic case for all. This would result in one very well designed system being duplicated to suit throughput requirements. In addition, it may be possible to create individual bespoke designs for parts of the process that have different requirements. As a result, a typically inaccurate ‘one size fits all’ approach can be replaced with targeted cells completing specific tasks that fit their design criteria more appropriately.

### 7.2.3 Influence of Constituent Materials

One of the most profound consequences of using reliable FE analysis to design reactor vessels is the ability to evaluate the influence alternative constituent materials will have on performance. Throughout this work Perspex and PVC were used due to necessity of transparent cell walls, availability and ease of fabrication. However, by replacing these with stiffer materials of greater acoustic impedance, one may be able to minimise the acoustic energy radiated into the vessel structure. This would, of course, alter the field structure but should result in greater acoustic intensities within the target region. Common materials that may be used to such effect would be
aluminium, steel, ceramic and glass. On the other hand, it may be desirable to aid the transmission of energy into the system in order to minimise the pressure doubling effects at boundaries. In this case a variety of polymers, possibly employed in a graded structure could be implemented.

Furthermore, the physical dimensions of the chamber can be easily altered to tailor the system to a particular requirement. This would allow the system designer to take into account what effects may occur of the system has to conform to specific dimensions in order to integrate into existing equipment.

### 7.2.4 Nonlinear Tonpilz Device

Perhaps one of the more unexpected outcomes of this work was the discovery that the Tonpilz devices used throughout are inherently nonlinear, most likely due to their construction. The centre bolt creates intimate mechanical contact between all components however the lack of bonding may result in a non-linear hammering action being facilitated as the device is actuated. Recently, Weidlinger Associates have announced an upgrade to the PZFlex code that will allow this type of bonding, or lack of, between elements to be modelled. Therefore, any harmonic content generated through the repeated collision of materials at the device resonant frequency could be modelled. This would provide insight into not only the behaviour of the Tonpilz device, but ultimately how these harmonics affect the transmission characteristics of these devices and how they may deviate from linear field predictions. It may also develop the possibility of these devices being utilised for more broadband applications and where harmonic content is desirable.
7.2.5 Simulating the Effects of Cavitation

It has been noted throughout this thesis that both the practical measurement and simulation of cavitation are troublesome problems to solve. NPL have prototyped a measurement device that monitors cavitation while the simulation difficulties remain. However, estimating the impact of the onset of cavitation is possible in the PZFlex code if some approximations are made. Treating regions of cavitation holistically, one can assume that this region will generally be subject to higher attenuation for any acoustic energy propagating through. Ignoring the scattering of acoustic energy for the present time, it would be possible to artificially increase the damping of a designated region via the material properties and, therefore, partially represent some of the effects of cavitation. Next, a region can be assigned a volumetric proportion of small voids in order to emulate the presence of microbubbles. This would require a 3D model otherwise symmetry would create rings of voids as opposed to spheres. Finally, a combination of these two separate models may yield an approximation of the field profile expected when cavitation has been initiated within the reactor.
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